# SECOND SEMESTER SYLLABUS

# M. Sc. MATHEMATICS

APPLICABLE TO BATCHES

2024 - 2026

2025 - 2027

2026 - 2028

BABA GHULAM SHAH BADSHAH UNIVERSITY RAJOURI, J&K, INDIA

## SECOND SEMESTER

| COURSE CODE | COURSE TITLE | NO. OF CREDITS | DISTRIBUTION OF MARKS | | |
|---|---|---|---|---|---|
| CORE COURSES | | | SA | UE | TOTAL |
| MM - 201 | Real Analysis - II | 4 | 40 | 60 | 100 |
| MM - 202 | Complex Analysis - II | 4 | 40 | 60 | 100 |
| MM - 203 | Functional Analysis | 4 | 40 | 60 | 100 |
| MM - 204 | Linear Algebra | 4 | 40 | 60 | 100 |
| MM - 205 | Techniques in Differential Equations | 2 | 20 | 30 | 50 |
| MM - 206 | Introduction to MATLAB (Lab. Course) | 2 | 25 | 25 | 50 |
| Choice based open elective course (Students are required to opt one course from a list of courses offered by different departments of the university at the beginning of semester second.) | | 4 | 40 | 60 | 100 |
| | TOTAL | 24 | 245 | 355 | 600 |

# SEMESTER - II

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|---|---|---|---|
| Course Title | Real Analysis - II | Maximum Marks | 100 |
| Course Code | MM-201 | University Examination | 60 |
| Credits | 4 | Sessional Assessment | 40 |
| | | Duration of Exam. | 3 HOURS |

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| Objectives …................... | | This course introduces the students the multivariable differential and integral calculus and Banach fixed point theorem with applications. |
| UNIT 01 | Total, Directional and Partial Derivatives of functions on Rn | Euclidean norm on Rn and its properties; Norm of a linear operator on Rn and its properties; Uniform Continuity of a linear operator on Rn; Openness of the set of all invertible linear opertaors on Rn; Total derivative of function at a point as a Linear operator on Rn, its uniqueness and relation with continuity; Derivative of Sum, difference, dot product and composition of two functions; Directional and Partial derivatives and their relationship with continuity and Total derivative. |
| UNIT 02 | Inverse and Implicit function Theorems | The Jacobian Matrix; Gradiant of a function; Mean Value theorem for differentiable functions and its consequences; Equality of Mixed partial derivatives; Taylor's formula for real valued functions on Rn; Functions with non zero Jacobian determinant - Image of an open ball containing an open ball, Image of an open set under an injection, injectivity of a function locally and openness of the image of an open set ; Inverse function theorem and Implicit function theorem. |
| UNIT 03 | Extrema of functions and Banach theorem with Applications | Extrema of functions - Second derivative test; Contraction Mapping and its continuity; Banach fixed point theorem; Applications of Banach theorem to Linear equations; Applications of Banach theorem to Differential equations - Picard existence and uniqueness theorem; Applications of Banach theorem to integral equations - Fredholm and Volterra integral equations. |
| UNIT 04 | Multiple Imtegrals | Definite integrals on rectangles - Lower & Upper sums, Integral of a function with elementary properties, Integrability of a continuous function, Iterated integrals, Fubini's Theorem; Definite integrals on regions of Type I and Type II; Integrals of functions of three or more variables; Change of Variables in Definite Integrals - Linear & Non linear change of variables, Polar coordinates, pherical coordinates. |
| UNIT 05 | Line and Surface Integrals | Line Integrals - dependence of line integral of a gradient vector field on end points, Green's theorem for rectangles, Green's theorem for Region of Type III; The cocept of surface integral - The Divergence theorem; Stoke's theorem. |

## COURSE OUTCOMES

On successful completion of this course, we expect that a student is able to explain

1 the concept of Euclidean norm and Norm of a linear operator on Rn with elementary properties

2 the concept of Total, Directional and Partial derivatives and their relationship with each other and continuity.

3 The concept of Jacobian matrix and properties of the functions with non zero Jacobian determinant and Taylor;s theorem.

4 Mean Value theorem for differentiable functions and its consequences and Equality of Mixed partial derivatives.

5 Explain the Extrema of functions along with Second derivative test and its applications.

6 Banach Contraction Mapping and its various applications.

7 Definite integrals on rectangles and regions of Type I and Type II and iterarted integrals with elementary properties.

8 Fubini's Theorem, Integrals of functions of three or more variables and Change of Variables in Definite Integrals.

9 the concept of Line and surface integrals.

10 the theorem of Green, the Divergence theorem and the Stoke's theorem

## Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

## BOOKS RECOMMENDED

### TEXT BOOKS

1. **Apostol, Tom M., (2002),** Mathematical Analysis, **1st edition, Narosa Publishing House.**

2. **Sloughter, D.(2024),** The Calculus of Functions of Several Variables, **LibreTexts.**

2. **Anton, H., Bivens, I. & Davis, S.(2012),** Calculus, **John Wiley and Sons.**

### REFERENCE BOOKS

1. **Carvin, L. T. & Szczarba, R.H. (2015)** Calculus in Vector Spaces, **Second Edition, CRC Press**

2. **Rudin, W., (1976),** Principles of Mathematical Analysis, **3rd edition, McGraw Hill International Edition.**

# SEMESTER - II

| | | | |
|---|---|---|---|
| Course Title | Complex Analysis - II | Maximum Marks | 100 |
| Course Code | MM - 202 | University Examination | 60 |
| Credits | 4 | Sessional Assessment | 40 |
| | | Duration of Exam. | 3 HOURS |

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| Objectives …................... | | The objective of this course is to introduce students to the fundamentals of Complex analysis (with applications) which is a tool with remarkable and almost mysterious utility in applied mathematics. |
| UNIT 01 | Integral of a complex function-I | Integral of a complex valued function of a real variable with elementary properties; Contour integral; ML-inequality; Simply connected domain; Cauchy-Goursat theorem; Deformation of contour; Fundamental theorems of integration - existance of antiderivative; Cauchy's integral formula for derivatives; infinite differentiability of analytic functions. |
| UNIT 02 | Integral of a complex function-II | Morera's theorem; Gauss mean value theorem; Maximum modulus principle for analytic functions; Cauchy' inequality; Liouville's theorem; Fundamental theorem of Algebra; The concept of Convex hull; Gauss & Lucas theorem concerning zeros of a polynomial; Weirstrass theorem for sequences of analytic functions. |
| UNIT 03 | Singularities, Zeros and Poles of an Analytic function (AF) | Taylor's and Laurent's series of an analytic function and their uniqueness; Taylor's & Laurant's theorem; Parseval's formula; The concept of singularities - Removable, Poles and Essential; Charecterization of zeros and poles of an AF in terms of an AF with no zeros; isolation property of zeros of a non zero AF; Schwartz Lemma; Reflection Principle; Riemann theorem; charecterization of poles & isolated singularities of an AF in terms of limit of modulus of the function. |
| UNIT 04 | RssidueTheorem and its applications | Residue of an AF at a point; Cauchy's residue theorem; Formula for calculating residue in terms of limit of derivatives; Formula for residue of quotient of two AFs; Casorati - Weirstrass theorem; Application of residue theorem to calculate ∫F(cosΘ, sinΘ)dΘ over the interval [0, 2pi]; The concept of meromorphic function; Argument principle; Winding number; Rouche's theorem. |
| UNIT 05 | Open mapping theorem and conformal mappings | Hurwitz theorem; The open mapping theorem; Non zero derivative of Univalent AF; Analyticity of inverse of a Univalent AF; Formula for summation of f(n) over integers of an AF f; The concept of conformal mapping; sufficient conditions for conformality of an AF; Critical points; magnification of angles at critical points; Bilinear transformation (BT) - existance of a unique BT which maps given three points on to another three points; Riemann mapping theorem (Statement only). |

## COURSE OUTCOMES

On successful completion of this course, we expect that a student is able to

1 Explain integral of a complex valued function of a real variable, Contour integral and the most valuable result of the course, namely, Cauchy-Goursat theorem.

2 Explain fundamental theorems of integration & Cauchy's integral formula &appreciate the result of infinite differentiability of analytic functions.

3 Explain basic and important results like Morera's theorem; Gauss mean value theorem; Maximum modulus principle Liouville's theorem, Gauss theorem, Lucas theorem & Weirstrass theorem for sequences of analytic functions.

4 Explain Taylor's theorem, Laurant's theorem and conncted concepts like the concept of singularities - Removable, Poles and Essential - and charecterize of zeros and poles of an AF in terms of an AF with no zeros.

5 Explain isolation property of zeros of a non zero AF and some important results like Schwartz Lemma, Reflection Principle, Riemann theorem.

6 Explain the concept of residue of an AF at a point and conncted results like Cauchy's residue theorem with applications - - calculation of ∫F(cosΘ, sinΘ)dΘ over the interval [0, 2pi].

7 Explain the concept of meromorphic function and conncted results like Argument principle, Winding number, Rouche's theorem and Hurwitz theorem.

8 Explain the open mapping theorem with its applications - analyticity of inverse of a Univalent AF.

9 Explain the concept of conformal mapping and related results and terminology.

## Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

## BOOKS RECOMMENDED

### TEXT BOOKS

1. **Mathews, J. H. & Howell, R. W., (2006),** Complex Analysis for Mathematics and Engineering, 5th edition, **B Jones and Bartlett Publishers .**

2.**Agarwal, R. P., Perara, K. & Pinelas, S.(2011),** An Introduction to Complex Analysis , **Spronger.**

### REFERENCE BOOKS

1. **Ahlofrs, L. R., (1996),** Complex Analysis, **McGraw Hill.**

2. **Brown, J. W. and Churchill, R. V., (2009),** Complex Variables and Applications, 8th Edition, **McGraw-Hill International.**

3. **Conway**, **J. B., (1973),** Functions of one Complex Variable, 2nd edition**, Springer International Student edition.**

4. **Rudin, W., (1987),** Real and Complex Analysis, 3rd edition, **McGraw Hill International Edition**.

# SEMESTER - II

| | | | |
|---|---|---|---|
| Course Title | Functional Analysis | Maximum Marks | 100 |
| Course Code | MM-203 | University Examination | 60 |
| Credits | 4 | Sessional Assessment | 40 |
| | | Duration of Exam. | 3 HOURS |

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|---|---|---|
| Objectives ….................. | | The main objective of this course is to introduce students to the fundamentals of functional analysis and make them aware of its applications. |
| UNIT 01 | Normed & Banach Spaces | The concept of norm on a vector space, standard examples and its basic properties; equivalence of norms on finite dimensional normed spaces (FDNS); completeness of FDNS; Riesz lemma; compactness of the closed unit ball and dimension of a normed space; relation b/w completeness and absolute convergence of a series; Banach spaces with standard examples; completion of a normed space. |
| UNIT 02 | Bounded linear operators on normed & Banach spaces | Definition, standard examples and elementry properties of bounded linear operators(BLO); continuity of linear operators on finite dimensional spaces; norm of a BLO; completeness of normed space of BLOs; dual spaces of R^n and l^p spaces, Zorn's Lemma (Statement Only) and its application to Hahn - Banach theorem for real & complex vector spaces; Hahn - Banach extension theorem for normed spaces and its various consequences. |
| UNIT 03 | Inner Product spaces, Hilbert spaces & Orthogonal complements | Definition and standard examples of inner product (IP) on a vector space (VS); Schwarz inequality; Hilbert space with standard examples; parallelogram law and polarization identity; orthogonal complement(OC) of a set and its preoperties; existence of minimising vector; projection theorem; OC of a closed subspace; charecterization of sets with dense spans in terms of OC. |
| UNIT 04 | Orthonomal sets in a Hilbert space & Dual of a Hilbert space | Definition and standard examples of orthonormal sets; Bessel's inequality; Fourier cofficients; Gram- Schmidt process; total orhonormal set (TONS); existance of a TONS in a Hilbert space; Hilbert dimension; Parsevel's relation; relation b / w separable Hilbert spaces & countability of TONS; isomorphic Hilbert spaces; Legender, Hermite and Laguerre polynomials; Riesz theorem; sesquilinear forms; Riesz representation for sesquilinear forms. |
| UNIT 05 | Reflexive spaces, convergence issues and fundamental theorems | Adjoint of a BLO b/w normed spaces; Reflexive spaces with standard examples - Hilbert spaces and finite dimensional normed spaces; relation b/w separability of a normed space and its dual; Uniform boundedness theorem and its application to space of polynomials and Fourier Series; strong and weak convergence; convergence of sequences of operators and functionsls; Open mapping and closed graph theorems. |

## COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 should be able to explain the concept norm on a vector space with its basic properties, Banach space with standard examples, convergence os a series in a normed space and compactness of the closed unit ball in a normed space.

2 should be able to explain the concept of a bounded linear operator on a normed space with standard examples and its connection with continuity, dual space with standard examples and Hahn - Banach theorem with its consequences.

3 should be able to explain the concept of inner product space, Hilbert space, polarization identity, its role in defining inner product, orthogonal complement of a set with its basic properties.

4 should be able to explain the existence and uniqueness of element of minimum norm and projection theorem.

5 should be able to explain the Bessel's inequality, Parseval's relation, total orthonormal set and its various charecterizations.

7 should be able to explain the concept of separable and reflexive normed spaces.

8 should be able to explain the concept of various types of convergences in normed spaces, space of bounded linear operators and in dual spaces.

9 should be able to explain closed graph theorem, open mapping theorem and Principle of uniform boundedness with applications.

## Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

# BOOKS RECOMMENDED

## TEXT BOOKS

1. **Kreyszig, E., (2006),** Introductory Functional Analysis with Applications, 1st edition, **Wiley Student edition.**

## REFERENCE BOOKS

1. **Bachman, G. and Narici, L., (1966),** Functional Analysis, **Academic Press New York.**
2. **Cheney, W., (2000),** Analysis for Applied Mathematics, **Springer.**
3. **Rynne, B. P. and Youngson, M. A., (2008),** Linear Functional Analysis, 2nd edition, **Springer.**
4. **Siddiqi, A. H., (2004),** Applied Functional Analysis, **Marcel-Dekker, New York.**

# SEMESTER - II

| | | | |
|---|---|---|---|
| Course Title | Linear Algebra | Maximum Marks | 100 |
| Course Code | MM-204 | University Examination | 60 |
| Credits | 4 | Sessional Assessment | 40 |
| | | Duration of Exam. | 3 HOURS |

**Objectives ….................** The main objective of this course is to familiarize the students to the fundamentals of linear algebra and their properties.

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|---|---|---|
| UNIT 01 | Vector Space and Linear Transformation | Recall of vector space; subspace; linear independence; linear dependence; basis; dimension and related properties; linear mapping; Matrix of a linear transformation; matrix of sum and composition of linear transformations; change of basis and transition matrices; similar matrices; permutation and its signature. |
| UNIT 02 | Spectral Theory | Eigen values and eigen vectors of a matrix and a linear transformation; algebraic and geometrical multiplicities of an eigen value; diagonalizable linear mapping/matrix; Cayley-Hamilton theorem; minimum polynomial of a matrix and its properties; invariant subspaces of a vector space; primary decomposition theorem (statement only) and its special cases; necessary and sufficient conditions for simultaneous diagonalization of two matrices. |
| UNIT 03 | Direct Sums of Subspaces | Direct sum; sum of subspaces; projection; idempotent; orthogonal complement; f-invariant; block diagonal form; primary decomposition; diagonalisation. |
| UNIT 04 | Nilpotency and Jordan Forms | Nilpotent linear transformations; existence of triangular matrix; Jordan decomposition theorem (statement only); index of a nilpotent linear transformation and its elementary properties; Jordan Block matrix; Jordan form; Jordan basis. |
| UNIT 05 | Bilinear and Quadratic Forms | Bilinear form; symmetric and skew symmetric bilinear forms; quadratic form; associated quadratic form; Sylvester theorem(statement only); positive definite quadratic form. |

## COURSE OUCOMES

On successful completion of this course we expect a student will be able to

1 explain vector space, linear dependence / independence, basis and dimension, linear transformation, change of basis matrix, permutation and its signature.

2 explain the concept of characteristic polynomial to compute the eigen values and eigen vectors of a square matrix and Cayley-Hamilton theorem.

3 explain the concept of minimum polynomial of a matrix and its properties, primary decomposition theorem and diagonalization.

4 explain the concept of direct sum, orthogonal complement, block diagonal form, primary decomposition.

5 explain the concept of Nilpotent linear transformations, Jordan decomposition theorem, Jordan Block Matrix, Jordan form, Jordan basis.

6 explain the concept of bilinear forms, symmetric and skew symmetric bilinear forms,

7 explain the concept of quadratic form and its properties.

## Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

# BOOKS RECOMMENDED

## TEXT BOOKS

1. **Blyth, T.S. and Robertson, E. F., (2007),** Basic Linear Algebra, 2nd Edition, **Spinger.**

2. **Blyth, T.S. and Robertson, E. F., (2008),** Further Linear Algebra, 2nd Edition, **Spinger.**

## REFERENCE BOOKS

1. **Hoffman , K. and Kunze, R.,(1971),** Linear Algebra, 2nd Ed., **Prentice-Hall, Inc., Englewood Cliffs, New Jersey.**

2. **Alexer , S. , (2014),** Linear Algebra Done Right, 3rd Ed., **Springer.**

3. **Herstein, I. N., (1975),** Topics in Algebra, **John Willy & Sons.**

# SEMESTER - II

| | | | |
|---|---|---|---|
| Course Title | Techniques in Differential Equations | Maximum Marks | 50 |
| Course Code | MM - 205 | University Examination | 30 |
| Credits | 2 | Sessional Assessment | 20 |
| | | Duration of Exam. | 2 HOURS |

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| Objectives ….................. | | The main objective of this course is to introduce students to the theory & techniques of solving various Ordinary Differential Equations. |
| UNIT 01 | Higher order linear differential equations - I | Basic existence theorem (Proof not included); basic theorems on linear homogenous equations; concept of Wronskian; reduction of order method; general solution of a homogeneous linear differential equation with constant coefficients. |
| UNIT 02 | Higher order linear differential equations - II | Method of undetermined coefficients; method of variation of parameters; Cauchy–Euler equation; power series about an ordinary point; singular point; method of Frobenius; Bessel's equation and Bessel's functions. |
| UNIT 03 | Sturm-Liouville boundary value problems | Sturm-Liouville problems; characteristic values; characteristic functions; orthogonality of characteristic functions; expansion of a function in a series of orthogonal functions; expansion problem; trigonometric Fourier series and its convergence. |

## COURSE OUTCOMES

On successful completion of this course, we expect that a student have understood

1 Basic existence theorem and other funcdamental theorems on linear homogenous equations.

2 the concept of Wronskia and is able to solve homogeneous linear differential equation with constant coefficients.

3 various methods such as method of undetermined coefficients and method of variation of parameters.

4 the method of solving Cauchy–Euler equation.

5 the method of Frobenius, methid of solving Bessel's equation and the concept of Bessel's functions.

6 in depth the Sturm-Liouville problems and trigonometric Fourier series with its convergence issues.

## Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 06 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 06 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 08 marks.

# BOOKS RECOMMENDED

## TEXT BOOKS

1. **Ross, S., (1984),** Differential Equations, **3 rd Edition, Wiley India (P) Ltd, New Delhi.**

## REFERENCE BOOKS

1. **Boyce,W. E., DiPrima, R.C., (2007),** Elementary Differential Equations and Boundary Value Problems**, 8 th edition, John wiley and sons.**

2. **Edward, P., (2005),** Differential Equation and Boundary Value Problems; **Computing and Modeling, 3 rd edition, Pearson Education.**

3. **Simmons, G. F., (2003),** Differential Equation with Applications and Historical Notes**, 2 nd edition, Tata McGraw Hill edition.**

# SEMESTER - II

| | | | |
|---|---|---|---|
| Course Title | Introduction to MATLAB (Lab. Course) | Maximum Marks | 50 |
| Course Code | MM - 206 | University Examination | 25 |
| Credits | 2 | Sessional Assessment | 25 |
| | | Duration of Exam. | 2 HOURS |

Objectives ….................... The aim of this Laboratory course is to introduce the students fundamental skills necessary to use MATLAB in academic research and engineering applications.

* Each student is required to maintain a practical record book.

* Two practical tests, one Internal and one External, are to be conducted.

* Each practical test will be of 25 marks.

* The student has to pass both internal and external practical tests separately scoring a minimum of 10 marks in each test

MATLAB Environment - Overview of MATLAB interface and workspace, Basic commands and syntax, Variables, data types, and workspace management; Matrix and Array Operations - Introduction to matrices and arrays; Basic matrix operations: addition, subtraction, multiplication; Element-wise operations and indexing; Basic Programming Constructs -writing MATLAB scripts, Control structures, defining and calling functions; Plotting and Data Visualization - Creating 2D plots, Customizing plots; Data Analysis and File I/O - Importing and exporting data, Working with built-in functions for statistical analysis,Data cleaning and manipulation.

## COURSE OUCOMES

On successful completion of this course, we expect that a student

1 Understands the MATLAB environment and its features.

2 Write simple MATLAB scripts and functions.

3 Perform basic operations with matrices and arrays.

4 is able to Work with scripts and functions.

5 Generate 2D plots and visualizations.

6 Apply MATLAB for basic data analysis tasks.

7 is able to explain Input/output commands.

8 is able to explain Relational and logical operators.