FOURTH SEMESTER SYLLABUS

M. Sc. MATHEMATICS

APPLICABLE TO BATCHES

2024 - 2026

2025 - 2027

2026 - 2028

BABA GHULAM SHAH BADSHAH UNIVERSITY RAJOURI, J&K, INDIA

FOURTH SEMESTER

COURSE CODE	COURSE TITLE	NO. OF CREDITS	DISTRIBUTION OF MARKS		
CHOICE BASED OPEN ELECTIVE COURSES			SA	UE	TOTAL
Elective - 01 (Choose any one of the following four courses)					
MM - 401	Advanced Topics in Topology	4	40	60	100
MM - 402	Tensor Analysis and Riemanian Geometry	4	40	60	100
MM - 403	Algebraic Topology	4	40	60	100
MM - 404	Modeling and Simulation	4	40	60	100
Elective - 02 (Choose any one of the following four courses)					
MM - 405	Hardy & Bergman Spaces	4	40	60	100
MM - 406	Algebraic Geometry	4	40	60	100
MM - 407	Approximation Theory	4	40	60	100
MM - 408	Complex Dynamics	4	40	60	100
Elective - 03 (Choose any one of the following four courses)					
MM - 409	Applied Functional Analysis	4	40	60	100
MM - 410	Banach Algebras	4	40	60	100
MM - 411	Algorithmic optimisation	4	40	60	100
MM - 412	Advanced Topics in Functional Analysis	4	40	60	100
Elective - 04 (Choose any one of the following four courses)					
MM - 413	Module Theory				
MM - 414	Commutative Algebra	4	40	60	100
MM - 415	Wavelets and Applications	4	40	60	100
MM - 416	Variational Inequalities With Applications	4	40	60	100
CORE COURSES					
MM - 417	Project / Seminar	8	0	200	200
	TOTAL	24	195	405	600

SEMESTER - IV

Course Title	Advanced Topics in Topology	Maximum Marks	100
Course Code	MM - 401	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives		The aim of this course is to introduce the students about some advanced topics of topology such as Filters, Nets, seperation axioms, compactification, metriziability and topological groups.
UNIT 01	Nets and Filters	Nets - definition and examples, subnets (SN), convergence & cluster point (CP) of a net, relation b/w CP of a net and a convergent SN, relation of convergent nets (CN) with closure & continuity, connection of CN in product spaces (PS) with projections,ultranet (UN), charecterization of compact sets in terms of nets and UNs; Filters - definition and examples, convergent filter (CF), CP of a filter, connection of CFs with closures & continuity, CFs in PS, ultra filter (UF), charecterization of compact sets in terms of filters and UFs; connection b/w nets and filters.
UNIT 02	T0, T1,T2, T3 & Tychnoff spaces	T0, T1, T2 ,T3 & Tychnoff spaces- definition, examples, their subspaces, their product & quotient spaces, their open / closed continuous images; Charecterization of T1 spaces in terms of closedness of singelton sets; charecterization of T2 spaces in terms of uniqueness of limits of nets / filters and closedness of the diagonal; charecterization of continuous open maps with Hausdorff range; Charecterization of T3 spaces in terms of nhood bases of closed sets; Sufficient condition for the range of continuous map on a T3 space in terms of openness & closedness of the map; Charecterization of Tychnoff spaces in terms of bounded real valued continuous functions and subspaces of a cube; Relation b/w T0, T1, T2 ,T3 & Tychnoff spaces.
UNIT 03	T4 & Lindeloff spaces	T4 & Lindeloff spaces - definition, examples, their subspaces, their product & quotient spaces, their open / closed continuous images; necessary condition for a space to be T4 in terms of the existance of a dense set and a closed, relatively discrete set; connection b/w compact Hausdorff and T4 spaces; Urysohn's lemma; Tiestz extension theorem; shrinkable open cover (SOC) - charecterization of T4 spaces in terms of SOC; Relation of Lindeloff with second countable and separable spaces; connection b/w T3 Lindeloff and and T4 spaces; relation b/w T0, T1, T2, T3, T4 and Lindeloff spaces.

UNIT 04	Compactification & Urysohhn's metrization theorem	One point compactification (OPC) of a topological space - its compactness, n/s conditions for its Hausdorffness; OPC of two homeomorphic spaces; The OPC of R and Sorgenfrey line; Compactification of a locally compact Hausdorff space (LCHS); Relationship b/w LCHS and Tychnoff spaces; Existance of compactification for a Tychnoff space; Stone-Cech Compactification; Metrizable spaces - Metriziability of product of a countably many metric spaces, Urysohn's metrization theorem, metriziability of the continuous image of a compact space in to a Hausdorff space.
UNIT 05	Topological Groups	Definition, examples and basic Properties of a Topological Group(TG); charecterization of topological groups; Product of a closed & compact set; Properties of closure of a set in a TG; Product of compact / connected/ locally connected sets; Symmetric sets in a TG; Product of TGs; Charecterization of open nhood base at identity; Subgroups in Topological Groups and their basic properties.

COURSE OUCOMES

On successful completion of this course, we expect that a student

1. Should be able to explain the concepts of nets, subnets, convergent net, relation of convergence of a net with continuity / closure of a set and relation of convergent nets with projection maps in a product space.
2. Should be able to explain the concepts of Filter, convergent filter, relation of convergence of a filter with continuity / closure of a set and relation of convergent filters with projection maps in a product space.
3. Should be able to explain connection between nets and filter.
4. Should be able to explain the concept of T0, T1, T2, T3, Tychnoff spaces, T4 spaces, Lindeloff spaces, relations between them and their various charecterizations.
5. Should be able to explain the subspaces, quotient spaces, continuous images of T0, T1, T2, T3, Tychnoff spaces, T4 spaces and Lindeloff spaces.
6. Should be able to explain the products of T0, T1, T2, T3, Tychnoff spaces, T4 spaces and Lindeloff spaces.
7. Should be able to explain the concept of compactification, one - point compactification and their various properties.
8. Should be able to explain metriziability of product of a countably many metric spaces, Urysohn's metrization theorem, and continuous image of a compact space in to a Hausdorff space.
9. Should be able to explain the concept of Topological groups and their various properties.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Willard, S., (1976),** General Topology **(1970), Dover Publications New York.**
2. **Husain, T., (1976),** Introduction to Topological Groups **(1994), Dover Publications, New York.**

REFERENCE BOOKS

1. **Adams, C. and Franzosa, R. (2009),** Introduction to Topology – Pure and Applied, **Pearson.**
2. **Munkers, J.R. ,(2000),** Topology, 2nd Edition, **PHI.**
3. **Patty, C.W. (2010),** Foundations of Topology, second Edition, Jones and Barlet.

SEMESTER - IV

Course Title	Tensor Analysis and Riemanian Geometry	Maximum Marks	100
Course Code	MM - 402	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …..................		The aim of this course to study fundamental ideas of tensors and their various types in detail.
UNIT 01	Tensors	Idea of differentiable manifolds with n-dimensions; space of n dimensions, subspaces; transformation of coordinates; scalar; contravariant (tangent) and covariant(cotangent) vectors; scalar product of two vectors; tensor space of rank more than one contravariant and covariant tensors; symmetric and skew-symmetric tensors; addition and multiplication of tensors; contraction; composition of tensors; quotient law; reciprocal symmetric tensors of the second order.
UNIT 02	Tensors and vectors	Riemannian space; fundamental tensor; length of a curve; magnitude of a vector; associated covariant and contravariant vectors; inclination of two vectors, orthogonal vectors; coordinate hypersurfaces; coordinate curves; field of normals to a hypersurface; principle directions for a symmetric covariant tensor of the second order; Euclidean space of n dimensions.
UNIT 03	Derivative of a vector and tensor	Levi-Civita tensors; Christoffell symbols and second derivatives; need for covariant derivative; parallel transformations; covariant derivative of a contravariant and covariant vector; curl of a vector and its derivative; covariant differentiation of a tensor; divergence of a vector.
UNIT 04	Geodesic	Gaussian curvature; Riemann curvature tensor; geodesics; differential equations of geodesics; geodesic coordinates; geodesic deviation; Riemannian coordinates; geodesic in Euclidean space; straight lines.
UNIT 05	Tensor and curvature	Parallel transport along an extended curve; curvature tensor; Bianchi identities; Ricci tensor; scalar curvature; killing vector field; space-time symmetries (homogeneity and isotropy); space time of constant curvature; conformal transformations.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1. Explain the concept of a tensor (with various operations such as addition, multiplication, composition) contravariant and covariant tensors, symmetric and skew-symmetric tensors, Levi-Civita tensors, Christoffell symbols,
2. Explain the idea of differentiable manifolds and contravariant (tangent) and covariant(cotangent) vectors.
3. Explain the Riemannian space, coordinate hypersurfaces and field of normals to a hypersur face.
4. Explain the principle directions for a symmetric covariant tensor of the second order.
5. Explain the covariant derivative of a contravariant and covariant vector and curl of a vector with its derivative.
6. Explain the covariant differentiation of a tensor and divergence of a vector.
7. Explain Gaussian curvature, Riemann curvature tensor, geodesics and its differential equations and coordinates.
8. Explain the Ricci tensor, space-time symmetries (homogeneity and isotropy), space time of constant curvature and conformal transformations.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Weatherburn, C. E., (1986),** An Introduction to Riemannian Geometry and Tensor Calculus , **Cambridge University Press.**

2. **Narlikar, J.V., (1978),** General Relativity and Cosmology, **The Mac-Millan Company of India Ltd.**

REFERENCE BOOKS

1. **Srivastava, S. K. & Sinha, P. K., (1998),** Aspects of Gravitational Interactions, **Nova Science publications Inc., Commack, NY.**

2. **Sokolnikoff, I. S., (1964),** Tensor Analysis, **I. S. John Wiley & Sons, Inc.**

SEMESTER - IV

Course Title	Algebraic Topology	Maximum Marks	100
Course Code	MS-403	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …..................		The aim of this course to study topology in algebraic context.
UNIT 01	Homotopy-I	Homotopy of paths; equivalence of path homotopy relation; product of paths and its basic properties; fundamental group of a topological space; homomorphism induced by continues path.
UNIT 02	Homotopy-II	Covering spaces; covering map examples; local homomorphism the fundamental group of circle; lifting of a map; lifting correspondence isomorphism between $S^1$and Z, retraction; non-retraction theorem; Brouwer fixed point theorem for the disc.
UNIT 03	Fundamental groups	Deformation retracts and homotopy type; the fundamental group of S^n; fundamental group of some surfaces; compactness of project plane; non commutatively of fundamental group of figure eight and double tores.
UNIT 04	Covering spaces-I	Equivalence of covering spaces; the general lifting lemma; relation between equivalent covering maps and conjugations of sub group; universal covering space; space without any universal covering space; existence of covering spaces; semi locally simply connected space.
UNIT 05	Covering spaces-II	Covering transformation; group of covering transformation; regular covering map; orbit space; the fundamental theorem of algebra; Borsule-Ulam theorem for S^2 the bisection theorem.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 Explain the concept of Homotopy of paths, their equivalence, product and various basic properties.

2 Explain the concept of fundamental group of a topological space and homomorphism induced by a continues path.

3 Explain the concept of a covering space, covering map examples, local homomorphism and the fundamental group of circle.

4 Explain some fundamental theorem such as non-retraction theorem and Brouwer fixed point theorem for the disc.

5 Explain the concept of Deformation retracts and homotopy type and the fundamental group of S^n with its basic properties such as non commutatively of fundamental group of figure eight and double tores.

6 Explain some fundamental theorems such as the general lifting lemma, the fundamental theorem of algebra, Borsule-Ulam theorem for S^2 and the bisection theorem.

7 Explain equivalence of covering spaces, relation between equivalent covering maps and conjugations of sub group, existence of covering spaces and semi locally simply connected space.

8 Explain the covering transformation, group of covering transformations and regular covering map.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Munkers, J.R. ,(2000),** Topology, 2nd Edition, **PHI.**

REFERENCE BOOKS

1. **Greenberg, J. M. and Harper, R. J., (1981),** Algebraic Topology: A First Course, **ABP.**

SEMESTER - IV

Course Title	Modelling and Simulation	Maximum Marks	100
Course Code	MM - 404	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives ………………..		The objective of this course is to study methods of model formation and their simulation techniques.
UNIT 01	Systems and models:	Definition and classification of systems; classification and limitations of mathematical models and relation to simulation; methodology of model building; modeling through differential equations; linear growth and decay models; nonlinear growth and decay models; compartment models.
UNIT 02	Monte Carlo simulation and case studies	Barterning model; basic optimization theory; Monte-Carlo simulation approaches to differential equations; local stability theory; classical and continuous models; case studies in problems of engineering and biological sciences.
UNIT 03	Real world models	Checking of model validity; verification of models; stability analysis; basic model relevant to population dynamics, ecology, environment biology through ordinary differential equations; partial differential equations and differential equations.
UNIT 04	Simulation of random variables	General techniques for simulating continuous random variables, simulation from normal and Gamma distributions, simulation from discrete probability distributions, simulating a non – homogeneous Poisson process and queuing system.
UNIT 05	Numerical method for continuous simulation	Basic concepts of simulation languages, overview of numerical methods used for continuous simulation, Stochastic models, Monte Carlo methods

COURSE OUTCOMES

On successful completion of this course, we expect that a student has understood

1 the concept and various classes and limitations of mathematical models and their relation to simulation.

2 the technique of modeling through differential equations with special focus on linear and non – linear growth and decay models.

3 the concept of Barterning model, classical and continuous models and Monte- Carlo simulation approaches to differential equations.

4 the methodology involved in case studies of problems of engineering and biological sciences.

5 The method of checking of model validity, verification of models and related stability analysis.

6 general techniques for simulating continuous and discrete random variables with emphasis on normal, Gamma, non – homogeneous Poisson distributions.

7 the basics of simulation languages and a general overview of numerical methods used for continuous simulation.

8 basics of Stochastic models and Monte Carlo methods.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Edward A. B., (2000),** An Introduction to Mathematical Modeling**, John Wiley.**

2 **Ross, S.M., (2012),** Simulation**, India Elsevier Publication.**

3. **Fowler, A. C., (1997),** Mathematical Models in Applied Sciences**, Cambridge University Press.**

REFERENCE BOOKS

1. **Fishwick, P.,(1995),** Simulation Model Design and Execution**, PHI.**

2. **Law, M., Kelton, W. D., (1991),** Simulation Modeling and Analysis**, McGraw- Hill**

3. **Geoffrey, G., (1982),** System Simulation**, PHI.**

4. **Kapoor, J. N., (1988),** Mathematical Modeling**, New Age.**

SEMESTER - IV

Course Title	Hardy and Bergman Spaces	Maximum Marks	100
Course Code	MM - 405	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …..................		The aim of this course to introduce some material related to two of the most important classes of Analytic functions - Hardy and Bergman spaces.
UNIT 01	Hardy Spaces on the Unit Disk - I	The concept of a Subhormonic function - subhormonicity of log(modulus of an analytic function) & pth power of modulus of an analytic function; Definition of Hardy spaces on the unit disk [$H^p(D)$]; Hardy theorem; Strict inequality satisfied by different Hardy spaces; $H^p(D)$ as a vector, normed and Banach space; Convergence of the series Summation(1-modulus of a_j), where a_js are seros of a $H^p(D)$ function; Factorization of an Hp(D) function in terms of (i) Blaschke product and a non vanishing Hp(D) function and (ii) in terms of Blaschke product and a non vanishing $H^2(D)$ function; $H^\infty(D)$ as an ideal in $H^p(D)$.
UNIT 02	Hardy Spaces on the Unit Disk - II	The function u_r of $L^p(\partial D)$ space corresponding to the function u of $H^p(D)$ space; Relation b/w L^p norm of u_r and Fourier cofficietns of u in $H^2(D)$; Redefinig the class $H^2(D)$ in terms of Fourier cofficients of its members; Existance of a function ũ in $L^2(\partial D)$ for a $H^2(D)$ function u such that limit of the norm of u_r-u as r goes to 1 is zero & its application to prove similar result for $L^p(\partial D)$ and $H^p(D)$ spaces; Cauchy integral type formula for u in H^2 (D) and then in $H^p(D)$ in terms of the corresponding ũ; Sufficient conditions for u of $H^p(D)$ to vanish in terms of ũ.
UNIT 03	Hardy - Hilbert Spaces on the Unit Disk	$H^2(D)$ as an innerproduct and Hilbert space; Boundedness of point evaluations on $H^2(D)$; Reproducing Kernels and their application to esbalish a relationship b/w convergence in H^2 (D) and convergence of a sequence of analytic functions; The function $(1-z)^{-s}$ $(0<s<0.5)$ as a member of $H^2(D)$ space; Applications of the reproducing kernels to prove Possion integral formula for a $H^2(D)$ function; $H^2(D)$ functions with boundary in $L^\infty(\partial D)$; Symmetric derivative of a complex valued function of real variable; Fatou's theorem (statement only) & its applications - relation b/w u_r & ũ for u in $H^2(D)$.

UNIT 04	Bergman spaces on Unit Disk	The Bergman space $A^2(D)$ of analytic functions as a vector, inner product and Hilbert space; Existance of a countable orthonormal basis in $A^2(D)$; Boundedness of point evaluations on $A^2(D)$; Existence of a sequences of polynomials converging to members of $A^2(D)$; The Bergman Kernel K and its various properties - its belongingness in $A^2(D)$ and uniqueness; Relation b/w an ONB of $A^2(D)$ and supremum of square of modlus of a $A^2(D)$ function; Bergman kernel as a series involving ONB; Explicit expression for Bergman kernel of $A^2(D)$; Introduction to $A^p(D)$ spaces.
UNIT 05	Composition Operators on Hilbert - Hardy spaces of the Unit disk	The definition of a composition operator; Boundedness of the Composition operator on H2(D) space; Little's wood sibordination theorem; Upper and lower bounds for norm of a composition operator; Charecterization of composotion operators in terms of reproducing kernels; Charecterization of composotion operators with the help of powers of z; Normal & Compact composition operators.

COURSE OUTCOMES

On successful completion of this course, we expect that a student is able to explain

1	The concept of a Subhormonic function,, Hardy spaces on the unit disk and its Banachness.
2	The factorization of an $H^p(D)$ function in terms of (i) Blaschke product and a non vanishing Hp(D) function and (ii) in terms of Blaschke product and a non vanishing H2(D) function.
3	Relation between members of $L^p(\partial D)$ spaces and the corresponding spaces $H^p(D)$ spaces.
4	The Hardy - Hilbert space with various examples of their members.
5	Reproducing Kernels, Fatou theorem and their applications.
6	The Bergman space A2(D) of analytic functions as a vector, inner product and Hilbert space, and its separability.
7	The Bergman Kernel K and its various properties.
8	The idea composition operator and its various properties.
9	Little's wood subordination theoremand various Charecterization of composotion operators.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1 **Greene, R. E. & Krantz, S.G.(2006),** Function Theory of One Complex Variable, **American Mathematical Society, Graduate Studies in Mathematics, Third edition, Volume 40.**

2 **Avendano, R. A. M. and Rosenthal, P.(2007),** An introduction to Operators on Hardy - Hilbert Spaces, **Springer.**

REFERENCE BOOKS

1. **Carnett, J. B.,(1981),** Bounded Analytic Functions, **Academic Press.**

2. **Hoffman, K., (2009),** Banach Spaces of Analytic Functions, **Prentice Hall Engle wook Cliffs, New Jersay.**

3. **Duren, P. L., (1970),** Theory of HP Spaces, **Academic Press.**

4. **Rudin, W., (1987),** Real and Complex Analysis, **3rd edition, McGraw Hill Book Co.**

4. **Hedenmalm, H., Korenblum, B. & Zhu. K. (2000),** Theory of Bergman Spaces, **Spronger.**

SEMESTER - IV

Course Title	Algebraic Geometry	Maximum Marks	100
Course Code	MM - 406	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …...................		The aim of this course to introduce the students by the concept of algebraic geometry and its applications
UNIT 01	Rational maps	Introduction; affine varieties, Hilbert's Null stellensatz, polynomial function and maps; rational functions and maps.
UNIT 02	Smoothness, singularity and dimension	Projective space; projective varieties; rational functions and morphisms; smoothpoints and dimension, smooth and singular points, algebraic characterizations of the dimension of a variety.
UNIT 03	Plane curves	Plane cubic curves, plane curves, intersection multiplicity, classification of smooth cubics, the group structure of an elliptic curve.
UNIT 04	Cubic surfaces	Cubic surfaces, the existence of lines on a cubic, configuration of the 27 lines, rationality of cubics.
UNIT 05	Theory of curves	Introduction to the theory of curves, divisors on curves, the degree of a principal divisor, Bezout's theorem, linear system on curves, projective embeddings of curves.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 Explain rational functions and maps, affine varieties and their properties.

2 Explain projective space and projective varieties and algebraic characterizations of the dimension of a variety.

3 Explain the plane cubic curves and intersection, multiplicity, classification of smooth cubics.

4 Explain the group structure of an elliptic curve.

5 Explain cubic surfaces and the existence of lines on a cubic.

6 Explain configuration of the 27 lines and the rationality of cubics.

7 Explain divisors on curves and the degree of a principal divisor

8 Explain the bezout's theorem and projective embeddings of curves.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Hulek, K. (translated by H. Verrill), (2003), Elementary Algebraic Geometry–Student Mathematical Library, vol 20,American Mathematical Society.

REFERENCE BOOKS

1. Hartshorne, R., (1977), Algebraic Geometry, Springer Verlag.

2. Harris, J., (1992),Algebraic Geometry: A First Course, Springer Verlag.

3. Elliptic Curves, Notes on NBHM Instructional Conference held at TIFR,(1991),Mumbai.

SEMESTER - IV

Course Title	Approximation Theory	Maximum Marks	100
Course Code	MM-407	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …................... The main objectives of this course is to familiarize the students with the fundamentals of approximation theory

UNIT 01	Existence and uniqueness of best approximations	Approximation in a metric space; approximation in a normed space; convexity and strict convexity; conditions for the uniqueness of the best approximation; best approximation operators and their continuity.
UNIT 02	Approximation by Algebraic & trigonometric polynomials	Bernstein operators and their basic propereties; Landau opeartors and with properties; modulus of continuity and its various properties; positive operators; Bohman-Korovkin Theorem; approximation by Trigonometric Polynomials; Weierstrass's Second Theorem.
UNIT 03	Characterization of Best Approximation	The best approximating constant; charecterization of best approximation in terms of alternating set; de La Vall´ee Poussin theorem; the monic polynomial of degree exactly n having smallest norm in C[a, b]; Chebyshev Polynomials and their various properties; uniform Approximation by Trig Polynomials.
UNIT 04	Interpolation	Vandermonde's determinant; Lagrange Interpolation; Newton's formula; Faber theorem; Lagrange formula with remainder; Lebesgue numbers; Lebesgue's Theorem; Chebyshev Interpolation; Hermite Interpolation; The Inequalities of Markov and Bernstein;
UNIT 05	Jackson's Theorems & Orthogonal Polynomials	Jackson's Theorems - dirct and Inverse; Orthogonal Polynomials with standard examples; least-squares approximations in inner product spaces; Leibniz's formula; The Christoffel-Darboux Identity.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 should be able to explain approximation in a metric and normed space, the concepts of convexity and strict convexity and uniqueness conditions for the best approximation.

2 should be able to explain the concept of best approximation operators and their properties.

3 Should be able to explain Bernstein operators, Landu operators and their basic properеties.

4 Should be able to explain modulus of continuity and its various properties, Bohman-Korovkin Theorem,

5 Should be able to explain the charecterization of best approximation in terms of alternating set and Chebyshev Polynomials with their various properties.

6 Should be able to explain the Vandermonde's determinant, Lagrange Interpolation, Lebesgue numbers & Lebesgue's Theorem.

7 should be able to explain Chebyshev Interpolation, Hermite Interpolation, the Inequalities of Markov and Bernstein.

8 should be able to explain various Jackson's Theorems & the concept and examples of Orthogonal Polynomials.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Carothers, N. L. , A Short Course on Approximation Theory, Department of Mathematics and Statistics, Bowling Green State UniversitY

2. Mhaskar, H. N, , Devidas V. Pai (2000),Fundamentals of Approximation Theory, CRC Press.

REFERENCE BOOKS

1. N. L. Carothers, A Short Course on Approximation Theory, Department of Mathematics and Statistics, Bowling Green State University.
2. P. P. Korovkin(1960), Linear operators and approximation theory, Hindustan Publishing Corporation, Delhi.
3. M J D Powell (1981), Approximation theory and methods, (CUP, reprinted 1988) .
4. E. W. Cheney (1982), An Introduction to Approximation Theory, 2nd ed., New York: Chelsea.
5. R. DeVore, G.G. Lorentz(1993), Constructive Approximation, Springer Verlag.
6. R Goldman (2002),Pyramid Algorithms, A Dynamic Programming Approach to Curves and Surfaces for Geometric Modeling, Elsevier.
7. Radu Paltanea (2004), Approximation Theory Using Positive Linear Operators, Birkhauser, Springer.

SEMESTER - IV

Course Title	Complex Dynamics	Maximum Marks	100
Course Code	MM-408	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives ………………		The aim of this course is to study fundamentals of complex dynamics –Conformal map, iterations of Rational functions in a plane; Fatou and Julia sets.
UNIT 01	Conformal mapping	Linear and reciprocal transformations; square map, conformal and isogonal maps; conformality theorem; Bi-linear transformation and its basic properties; fixed points of a bilinear transformation; cross ratio; exponential and trigonometric transformations; Riemann mapping theorem (proof not included).
UNIT 02	Iterations of functions and various metrices on $\mathbb{C}_\infty$	Iteration of a Mobius transformation; attracting, repelling and indifferent fixed points; iterations of $R(z) = z^2$; the extended complex plane; chordal metric; spherical metric; relation between chordal metric and spherical metric.
UNIT 03	Conjugacy classes of rational maps	Rational maps; Lipschitz condition; conjugacy classes of rational maps; valency of a function; fixed points; critical points; Riemann Hurwitz relation.
UNIT 04	Equicontinuity and Normality	Equicontinuous functions; normality sets; Fatou sets and Julia sets; completely invariant sets; normal families and equicontinuity.
UNIT 05	Fatou and Julia sets	Properties of Fatou and Julia sets; exceptional points; backward orbit; minimal property of Julia sets; completely invariant components of the Fatou set; the Euler characteristic.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 Explain the concepts of repelling points, attracting points and indifferent fixed points.

2 Explain the concept of extended complex plane, chordal metric, spherical metric and relationship between chordal and spherical metrics.

3 Explain the concepts of conjugacy class of a rational map, valency of a function, completely invariant sets, normal families and equicontinuous family of functions.

4 Explain the cocept of exceptional points, backward orbit and minimal property of Julia sets .

5 Explain the concepts of Fatou sets, Julia sets and relationship between them.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Beardon, A. F.(1991),** Iteration of Rational Functions, **Springer Verlag**, **New York.**

2. **Carleson, L. and Gamelin, T . W., (1993),** Complex Dynamics, **Springer Verlag**.

REFERENCE BOOKS

1. **Hua, X. H., Yang, C. C., (2000),** Dynamics of Transcendental Functions, Gordan and Breach Science.

2. **Livi, R., Nadal, J. P. and Packard, N., (1993),** Complex Dynamics, **Nova Science Publication, Inc.**

3. **Morosawa, S., Nishimura, Y., Taniguchi, M., Ueda, T., (2000),** Holomorphic Dynamics, **Cambridge University Press.**

SEMESTER - III

Course Title	Applied Functional Analysis	Maximum Marks	100
Course Code	MM - 409	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives ………………..		The main objective of this course is to introduce students Distribution theory ,Sobolev spaces, Schwartz spaces and their applications.
UNIT 01	Distribution theory	Definition and examples of a test function; convergence in the space D(IRn) of test functions; definition and examples of distributions---regular; Dirac delta; Heaviside distribution; derivative of a distribution; convergence of distributions; product of a c∞(IRn) function and a distribution.
UNIT 02	Distribution theory and fundamental solution of Laplacian	Convolution of a test function and a distribution; differential operators--- fundamental solution of the Laplacian operator; Support of a distribution--- partitions of unity; as a subset of ;convolution of two distributions.
UNIT 03	Schwartz and Sobolev spaces	Schwartz space; tempered distribution and its Fourier transform; definition and first properties of Wk, p (Omega), completeness; embedding of W1,2® in to W0, infinity (R); other embedding theorems(without proofs); denseness of C infinity functions Wk, p(Omega).
UNIT 04	Sobolev spaces continued	Definition and first properties of general Sobolev spaces; completeness; distribution with compact support as an element of the general Sobolove space; dual of the general Sobolov space; Sololeve's embedding theorem; density and trace theorems(without proofs); Poincare's and convexity inequalities(without proofs).
UNIT 05	Symmetric and non-symmetric variational problems	Formulation of symmetric variational problems- bilinear forms; Ritz-Galerkin approximation problem; fundamental Galerkin orthogonality; Ritz method; formulation of non-symmetric variational problems-Galerkin approximation problem; Lax- Milgram theorem ;Cea's theorem; variational formulation of Poisson's equation and pure Neumann boundary value problems; boundary value problems in science and technology.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 The concept, examples and properties of test function and distributions(such as regular, Dirac delta and Heaviside)

2 The concept of derivative of a distribution, convergence of a sequence of distributions and product of a c∞(irn) function with a distribution.

3 The concept of convolution of a test function and a distribution, fundamental solution of the Laplacian operator, Support of a distribution, partitions of unity and convolution of two distributions.

4 The concept of tempered distribution and its Fourier transform.

5 Fundamental results such as Sololev's embedding theorem, density and trace theorems, Poincare's and convexity inequalities.

6 Formulation of symmetric and non – symmetric variational problems

7 The Lax- Milgram theorem, Cea's theorem and variational formulation of Poisson's equation and pure Neumann boundary value problems

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Berner, S. C. And Scott, L. R.,(2000), The Mathematical Theory of Finite element methods, 3rd edition, Springer.**

2 . **Cheney, W., (2000), Analysis for Applied Mathematics, Springer, New York.**

REFERENCE BOOKS

1. **Dautary, R. and Lions, J. L., (2000),** Mathematical Analysis and Numerical Methods for Science and Technology, **Vol. 2 Function and variational Methods, Springer, Berlin, Heidelberg, New York**

2. **Chipot, M., (2000),** Elements of Non Linear Analysis, **Birkhauser, Basel, London, Boston.**

3. **Siddiqi, A. H., (2004),** Applied Functional Analysis, **Marcel-Dekker, New York.**

SEMESTER - IV

Course Title	Banach Algebras	Maximum Marks	100
Course Code	MM-410	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives		The aim of this course to study Banach algebra and its spectral theory.
UNIT 01	Banach Algebra and its spectral properties	Definition, examples and elementary properties of Banach Algebra; ideals in a Banach algebra; properties of set of invertible elements of a Banach algebra; properties of maximal ideas of a Banach algebra; quotient space of a Banach algebra; spectral of an element of a Banach algebra; formula for calculating spectral radius.
UNIT 02	Spectral and Riesz functional calculus	Riesz functional calculus and its uniqueness; spectral mapping theorem; dependence of the spectral on the algebra; spectral of a linear operator; approximate point spectral of a linear operator.
UNIT 03	Abelian Banach algebra and C* algebra	Gelfand - Mazur theorem; maximal ideal space of a Banach algebra and its properties; Gelfand transforms and its properties; radical of a Banach algebra; definition, examples and elementary properties of C* algebra; Abelian C* - algebra and the functional calculus in C* - algebra.
UNIT 04	C* - algebra – I	Hermitian elements; positive elements in C* - algebra; space of positive elements and their properties; polar decomposition; ideas and quotients in C*-algebra; representation of a c* - algebras; cyclic representation; state of a c*-algebra; Gelfand – Naimark – Segal construction.
UNIT 05	C*-algebra – II	Spectral measures; WOT; SOT; spectral theorem; topologies on B(H); commuting operators; double commutant theorem; Fuglede – Putnam theorem; Abelian Van Neumann algebras.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1. Explain the concept, examples and properties of Banach Algebra, quotient space of a Banach algebra and the set of invertible elements of a Banach algebra.
2. Explain the concept of ideals and maximal ideas of a Banach algebra.
3. Explain the concept of spectrum of an element of a Banach algebra and formula for calculating spectral radius.
4. Explain Riesz functional calculus and its uniqueness, spectral mapping theorem and dependence of the spectral on the algebra.
5. Explain Gelfand - Mazur theorem, Gelfand transforms and its properties, radical of a Banach algebra and maximal ideal space of a Banach algebra with its properties.
6. Explain the concept and elementary properties of C* algebra, Abelian C* - algebra, functional calculus in C* - algebra, positive elements in C* - algebra and their space with properties.
7. Explain the concept of representation of a c* - algebra, state of a c*-algebra, Gelfand – Naimark – Segal construction and Abelian Van Neumann algebra.
8. Explain some fundamental theorems such as double commutant theorem and Fuglede – Putnam theorem.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Conway, J. B., (2008), A Course in Functional Analysis, 2nd edition, Springer.

REFERENCE BOOKS

1. Douglas, R. G., (2008), Banach Algebra Techniques in Operator Theory, 2nd edition, Springer.

2. J.M.G. Fell and R.S. Doran (1988), Representation of *-Algebras, Locally Compact Groups and Banach * Algebraic Bundles, Vol I, II, Academic Press.

SEMESTER - IV

Course Title	Algorithmic Optimisation	Maximum Marks	100
Course Code	MM - 411	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …..................		The objective of this course is to introduce students to solve different problems by using optimization techniques.
UNIT 01	Calculus in normed spaces	Frechet derivative --- definition, examples, chain rule, mean value theorem, implicit function theorem (proof not included); second derivative; Taylor's formulae for first and second derivatives; Gateaux derivative and its relation with Frechet derivative.
UNIT 02	Extrema of real valued functions on normed spaces	Euler's equation; necessary conditions for constrained relative extremum Lagrange's multiplier ; necessary and sufficient condition for relative minimum in terms of second derivative; convex functions and their relation with first and second derivatives ; convexity and relative minima; Newton's method(convergence proofs not included).
UNIT 03	General results on optimization problems	Gradient of a functional on a Hilbert space; special classes of optimization problems; existence of solution of optimization problem for coercive and quadratic functional; examples of optimization problems; relaxation and gradient methods for unconstrained problems(convergence proofs not included) properties of elliptic functional; conjugate gradient method for unconstrained problems (convergence proofs not included).
UNIT 04	Non linear programming	Relaxation and gradient penalty–function methods for constrained problems (convergence proofs not included); introduction to non-linear programming problems; Farkas lemma ; Kuhan-Tuckar conditions – cone of feasible directions and its important properties; necessary and sufficient condition for the existence of a minimum in non-linear programming; properties of saddle points; Lagrangian associated with the optimization problems; duality; Uzawa's method (convergence proofs not included).
UNIT 05	Linear Programming	General results on linear programming; examples of linear programming problems; the simplex method – polyhedron and its properties; duality and linear programming – necessary and sufficient conditions for the existence of a minimum in linear programming; duality in linear programming; Lagrangian; relation between duality and simplex method.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1. the concept of Frechet derivative, Gateaux derivative and relation between them.
2. basic results such Mean value theorem, chain rule, implicit function theorem, Taylor's formulae for first and second derivatives.
3. the concept of relative extremum, the Lagrange's multipliers and the Euler's equation.
4. the concept of convex functions and their relation with first and second derivatives.
5. the concept of Gradient of a functional on a Hilbert space, elliptic functional, saddle points, Lagrangian associated with the optimization problems, duality and classification of optimization problems.
6. the basic results connected with the existence of solution of optimization problem for coercive and quadratic functional, relaxation, gradient and conjugate gradient methods for unconstrained problems.
7. Basic results connected with non – linear Programming such as Farkas lemma, Kuhan-Tuckar conditions, necessary and sufficient condition for the existence of a minimum, Uzawa's method.
8. the basic results on linear programming, the simplex method, necessary and sufficient conditions for the existence of a minimum and the relation between duality and simplex method.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Ciarlet, P. G., (1989), Introduction to Numerical Algebra and Optimization, Cambridge University Press Cambridge.**

REFERENCE BOOKS

1. **Cheney, W., (2001),** Analysis for Applied Mathematics, **Springer**, **New York.**

2. **Neunzert, H. and Siddiqi, A. H., (2000),** Topics in Industrial Mathematics-case Studies and Related Mathematical Methods, **Kluwer Academic Publishers, Dordrecht, Boston, London.**

3. **Polok, E., (1997),** Optimization Algorithms and Consistent Approximations Applied Mathematical Sciences Series, **Springer, New York.**

SEMESTER - IV

Course Title	Advanced Functional Analysis	Maximum Marks	100
Course Code	MM - 412	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …....................		The aim of this course to study advance topics of functional analysis.
UNIT 01	Topological vector spaces (TVS	Definition and examples of topological vector spaces; convex and absorbing sets; translation and multiplication operators; local base in a TVS; types of TVS; separation properties; simple properties of closure and interior in TVS.
UNIT 02	Linear transformations	Continuity of linear mappings; finite dimensional spaces; relation between LCTVS and its dimension; metrization; relation between F-space and closed subspace of a TVS; bounded linear transformations; semi norm and local convexity; properties of semi norm sets; MinKowski's functional and its properties.
UNIT 03	Fundamentals theorems and special spaces	Necessary and sufficient condition for a TVS to be normable; quotient spaces of a TVS; semi norm and quotient spaces; the spaces $C(\Omega)$, $H(\Omega)$; $C^{\infty}(\Omega)$ and Q_k , $L^p(0<P<)$; equicontinuity; Banach – Steinhaus theorem; continuity of limits of sequences of continuous linear mappings; open mapping theorem and its corollaries.
UNIT 04	Some fundamental theorems	Closed graph theorem; bilinear mappings; dual space; Hahn-Banach separation theorem and its various corollaries; the weak topology of a TVS; the weak* topology of dual space of a TVS; Banach- Alaogule theorem and its applications.
UNIT 05	Convexity	Convex Hull of a subset of a TVS and its properties; extreme points; the Krein- Milman's theorem; Milman's theorem; polar; bipolar theorem; Barelled and Bornological spaces; semi reflexive and reflexive topological vector spaces.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1. Explain the concept and examples of topological vector spaces(TVS), convex and absorbing sets, local base in a TVS and Locally convex TVS with its relation with dimension of the space.

2. Explain the separation properties in a TVS and the concept of closure and interior in a TVS.

3. Explain the concept and properties of continuity of linear mappings and relationship between F-space and closed subspace of a TVS.

4. Explain the concept of semi norm, its various properties and minkowski's functional.

5. Explain some Fundamental theorems such as Banach – Steinhaus theorem, open mapping theorem (with consequences), Closed graph theorem, Hahn-Banach separation theorem (with corollaries), Banach- Alaogule theorem (with applications), the Krein- Milman's theorem, Milman's theorem and bipolar theorem.

6. Explain the necessary and sufficient condition for a TVS to be normable and quotient spaces of a TVS.

7. Explain the spaces C(Ω) , H(Ω); C∞(Ω) and Qk , Lp(0<P<1) and the continuity of limit of sequence of continuous linear mappings.

8. Explain the concept of bilinear mappings, the weak and weak* topology, Convex Hull (with properties), extreme points, Barelled and Bornological spaces, semi reflexive and reflexive topological vector spaces.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Rudin, W., (1973), Functional Analysis, Tata Mcgraw Hill.

REFERENCE BOOKS

1. Schwartz, L., (1975),Functional Analysis, Courant Institute of Mathematical Sciences.
2. Treves, F. (1967), Topological Vector spaces, Distributions and Kernels Academics Press.
3. Kothe, G. (1976), Topological Vector Spaces-II, Springer Verlag, New York.
4. Larsen, R., (1972), Functional Analysis, Marcel Dekker.

SEMESTER - III

Course Title	Module Theory	Maximum Marks	100
Course Code	MM-413	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …..................		The aim of this course to study modules and their properties.
UNIT 01	Fundamentals of modules	Left modules and right modules; examples of modules; submodules; intersection, union and sum of sub-modules of a module; finitely generated module; homomorphism; fundamental theorems on homomorphism and quotient modules.
UNIT 02	Free modules-I	Direct sum of modules; equivalent condition for direct sum; free modules; characterization of free modules; cardinality basis of a module; rank of finitely generated free module; simple and semisimple modules.
UNIT 03	Free modules-II	Finitely generated free module over PID; the invariant factor decomposition; structure theorem for finitely generated modulus over a PID; torsion module and torsion free module; condition for a finitely generated module over a PID to be free module; the primary decomposition theorem; Chinese remainder theorem.
UNIT 04	Projective and injective modules	Exact sequences; projective modules; characterization of projective modules; condition for a ring to be semi-simple ring; injective modules; characterization of injective modules; Baer's criterion; injective hall; Northerian rings; necessary and sufficient condition for a ring to be Neotherian ring;
UNIT 05	Simple rings	Simple ring; Schin'slemma; semi-simple modules; the Astin-Wedder Burn theorem; simple modules; Jacobson radical; Astinan ring; Hopkins Levitzki theorem.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 explain the concept, examples and basic properties of modules, submodules, quotient modules, simple and semi - simple modules.

2 explain the fundamental theorems on homomorphism between modules.

3 explain the concept of free modules (its various characterizations) and rank of finitely generated free modules.

4 explain the concepts of Finitely generated free module over PID, torsion module and torsion free module and the invariant factor decomposition.

5 explain some fundamental results such as structure theorem for finitely generated modulus over a PID, condition for a finitely generated module over a PID to be free module, the primary decomposition theorem and Chinese remainder theorem.

6 explain the concepts, examples and properties of projective and injective modules.

7 Explain the concept , examples and properties of Simple ring, Northerian rings and semi-simple modules.

8 Explain some fundamental results such as condition for a ring to be semi-simple ring, necessary and sufficient condition for a ring to be Neotherian ring, Baer's criterion, Schin's lemma, Astin-Wedder Burn theorem, Hopkins Levitzki theorem.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Grillet, P. A., (2007),** Abstract Algebra: Graduate Texts in Mathematics, **2nd edition, Springer.**

REFERENCE BOOKS

1. **Blyth, T.S., (1982),** Module Theory: An Approach to Linear Algebra, **Oxford University Press.**

2. **Albu, T., Birkenmeier, G.F., Erdogan, A. and Tercan, A., (2010),** Rings and Module Theory, **Birkhäuser Basel.**

SEMESTER - IV

Course Title	Commutative Algebra	Maximum Marks	100
Course Code	MM - 414	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …...................		The aim of this course to study ideals, modules and rings.
UNIT 01	Ideals	Ring, ring homomorphism, ideals, operation on ideals, quotient rings, zero-divisors, nilpotents and units, prime and maximal ideals, local ring, Nilradical and Jacobson radical, exercises based on above topics.
UNIT 02	Modules	Module homomorphism, Submodules, Quotient modules, Operation on submodules, direct sum and product of modules, Finitely generated modules; Nakayama lemma, Tensor product of modules, Exercises based on the above topics.
UNIT 03	Localization and decomposition	Localization properties of localization, primary decomposition; primary ideals, uniqueness of primary decomposition, exercises based on above topics.
UNIT 04	Integral dependence	Integral dependence; transitivity of integral dependence, going-Up and going down theorems, exercises based on above topics.
UNIT 05	Noethrianrings	Chain condition; Noetherian and Artinian modules, Noetherian rings; Hilbert basis theorem, irreducible ideals and primary decomposition in Noethrian rings, exercises based on above topics.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 Explain the concept, examples and fundamental properties of Ring, ring homomorphism, ideals, quotient rings, zero-divisors, nilpotents and units, prime and maximal ideals, local rings, Nilradical and Jacobson radicals.

2 Explain the concept, examples and properties of module, Module homomorphism, Sub - modules, Quotient modules, direct sum and product of modules, Finitely generated modules and Tensor product of modules.

3 Explain the fundamental theorems such as Nakayama lemma.

4 Explain the concept and properties of Localization and primary decomposition.

5 Explain the concept and properties of Integral dependence, transitivity of integral dependence.

6 Explain the some fundamental theorems such as going-Up and going down theorems, Hilbert basis theorem.

7 Explain the concept of Noetherian and Artinian modules, Noetherian rings, irreducible ideals and primary decomposition in Noethrian rings.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Atiyah, M. f. and Macdonald, I. G.,(1994), Introduction to Commutative Algebra, Addision-Wesley Publishing Company.

REFERENCE BOOKS

1. Eisenbud, D., (1999), Commutative Algebra; With a View Toward Algebraic Geometry Springer- Verlag, New York.,

2. Kunz,E. (1985), Introduction to Commutative Algebraic Geometry, Birkhauser. Reid, M. (1996), Undergraduate Commutative Algebraic: London Mathematical Society Student Texts, Cambridge University Press, Cambridge.

SEMESTER - IV

Course Title	Wavelets and Applications	Maximum Marks	100
Course Code	MM - 415	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives ….................		This course is introduced taking into account its wide application and acceptance among researchers of various domains. This, in true sense, will promote inter-disciplinary studies and help students in job market.
UNIT 01	Multiresolution Analysisi	Definition and examples of multiresolution analysis (MRA); dilation equation, Mother wavelet, Haar wavelets, orthonormality of translates of an L^2 (R) function, filters, filter equality, scaling identity, representation of the filter m_gfor g∈W_0, Mother wavelet theorem
UNIT 02	Scaling functions	Compactly supported scaling functions φ, Properties of m_φ, Fourier transform of scaling functions, non sufficiency of trigonometric polynomials to generate wavelets, sufficient conditions for orthonormality of translates of scaling functions, Shannon wavelets, Riesz basis, characterization of Riesz basis, Riesz MRA, construction of an orthonormal basis from a Riesz basis.
UNIT 03	Frames	Franklin wavelets, Frames- tight and exact, Frame operator and its properties, Dual frame and its properties, equivalence of exact frames and Riesz basis in separable Hilbert spaces, Weyl- Heisenberg frames and their generation, splines and their basic properties.
UNIT 04	Continuous Wavelet Transform	Disadvantages of Fourier transform, Window function, centre, radius and width of a window function, windowed Fourier transform, Gabor transform and Short- Time Fourier transform, The uncertainty principle, basic wavelets and their examples, Continuous wavelet transform and its basic properties, Parseval's formula, reconstruction formula, Discrete wavelet transform, numerically stable recovery of a function through it DWCs.
UNIT 05	Applications	Introduction to applications of wavelets to Numerical Analysis – ODE and PDE; Signal analysis – audio compression, image and video compression, JPEG 2000, Texture classification, de-noising, finger prints; audio applications - audio structure decomposition, speech recognition, speech enhancement, audio de-noising.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

1 the concepts and various examples of Multiresolution Analysis (MRA), filters and wavelets.

2 various equations, identities and results such as dilation equation, filter equality, scaling identity and Mother wavelet theorem.

3 the concept and properties of scaling functions and Riesz basis.

4 the concept of a Frames and various terms connected with it such as tight and exact frame, Frame operator, Dual frame and Weyl- Heisenberg frame.

5 various transforms such as Fourier transform, Windowed transform, Gabor transform, Short- Time Fourier transform, wavelet transform and advantageous of one over the other.

6 fundamental results connected with wavelet transform such as Parseval's formula, reconstruction formula etc.,

7 the concept of Discrete wavelet transform and the numerically stable recovery of a function through its DWCs.

8 importance of wavelets in various areas of mathematics and other sciences such as Numerical Analysis, Signal analysis etc.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Bachman. G, Narici. L, Beckenstein. E (2010), Fourier and Wavelet Analysis, Springer

2. Siddiqi, A. H., (2004), Applied Functional Analysis, Marcel-Dekker, New York.

REFERENCE BOOKS

1. Daubechies, I.,(1992), Ten Lectures on Wavelets, CBS-NSF Regional Conferences in Applied Mathematics, 61, SIAM, Philadelphia, PA.

2. Hernandez, E., and Weiss, G.,(1996), A First Course on Wavelets, CRC Press, New York.

3. Teolis, A., (1998), Computation Signal Processing with Wavelets, 1st edition, Birkhauser, Boston, Basel.

SEMESTER - IV

Course Title	Variational Inequalities with Applications	Maximum Marks	100
Course Code	MM - 416	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives …................. The aim of this course is to introduce the students the modelling of real world problems through variotional inequalities.

UNIT 01	Minimization of convex functionals	Fundamental theorem; the general problem of minimization; the search of good hypothesis; minimization of quadratic functional; variational formulation of a minimization problem; variational inequalities; variational equation; projection on convex sets; projection Operators.
UNIT 02	Variational Inequalities	Fundamental theorem; bilinear forms; minimization and variational problems; Lions-Stampacchia theorem; Gateaux derivative; functionals defined on Hilbert spaces; Gateaux differential; G-derivative of a quadratic functional; general results on the equivalence of minimization and variational problems; indicatrix functions; fundamental result of existence and uniqueness;the concept of sub – differentials; right and left derivatives and subdifferential; multi-valued equations.
UNIT 03	Variational problems in one dimensions	Variational formulation of the obstacle problem; interpretation of the variational problem as a boundary value problem; equivalence of the two formulations of the obestacle problem; results of regularity; some considerations regarding second order linear problems; variational formulation of a boundary value problem; more general operators; eigenvalue problems.
UNIT 04	Differential operators	General comments on differential operators; differential operators -quasi-linear, semi-linear and linear; differential equations - quasi-linear, semi-linear and linear; classification of second order differential operators; some classical operators; initial value problems and boundary value problems; Cauchy problem; the operator .
UNIT 05	Linear problems	Variational formulation of the homogeneous Dirichlet problem on the coerciveness of the bilinear form; the Riesz-Fredholm theorem(statement only);general formulation in variational terms - the data and variational problem; intrepretation of the variational problem; examples of boundary value problems-the data, the differential operator, decomposition of the Laplace operator; examples - Dirichlet problems, Neuman problems, non-homogeneous Neuman problem, problem for Laplace operator.

COURSE OUTCOMES

On successful completion of this course, we expect that a student has understood

1. the concept of a convex functional, the general minimization problem and variational formulation of minimization problem.
2. the concept of projection Operators and their properties.
3. the concept of a bilinear form, Gateaux derivative, indicatrix function and sub – differential.
4. fundamental results on the equivalence of minimization and variational problems and existence and uniqueness.
5. the variational formulation of the obstacle problem, its interpretation as a boundary value problem and some connected results.
6. the concept of a differential operator, its various types such as quasi-linear, semi-linear and linear and classification of second order differential operators.
7. Variational formulation of various problems such as Cauchy problem, Dirichlet problem, Neuman problems and non-homogeneous Neuman problem.
8. some basic results such as Lions-Stampacchia theorem and Riesz-Fredholm theorem.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Baiocchia, C. and Capalo, A., (1984),** Variational and Quasi Variational Inequalities, Application to Free Boundary Problems**, John Wiley and sons Chechter.**

REFERENCE BOOKS

1. **Durant, G. and Lions, J.L.,(1976),** Inequalities in Mechanics and Physics**, Berlin, New York, Spring-Verlag**

2. **Kikuchi, N. and Oden, J.I, (1988),** Contact Problems in elasticity: A Study of Varitional Inequalities and Finite Element Methods, **Philadelphia, SIAM.**

3. **Lehrer, D., Kinder, S. and Stampacchia, G., (1980),** Introduction to Variational Inequalities, **Academic Press, New York.**

SEMESTER - IV

Course Title	Project / Seminar	Maximum Marks	200
Course Code	MM - 417		
Credits	8		

Objectives …..................... The objective of this course is to give a glimpse of research methods to the students.

* Each student has to submit a project report (and give a presentation of the same) on a topic of his/her own choice under the supervision of a teacher selected as guide by the student's choice from the Departmental faculty or under the joint supervision of a teacher from the Department and an appropriate member from any other Department or industry but after the permission of the Departmental guide.

* The marks by external examiner will be assigned on the basis of the project report submitted by the student, presentation and the viva-voce.

* The breakup of the marks for the project report (In the form of a dissertation) , presentation and the viva-voce marks is as follows:

	Dissertation	Presentation	Viva - Voce	Total
External Examiner	100	25	25	200

COURSE OUTCOMES

After a student completes the Major project, we expect a student have understood.

1 The method of searching literature, on a particular topic, form the internet.

2 The various potential areas of research, in a particular field, that can lead to a research degree(M. Phil/ Ph. D).

3 Various ethics of good research.

4 How to read a research paper and present it in his / her own words.

5 The use of various concepts from different courses for studying a research paper.

6 How to employ the skills learned through different courses to simplify complicated situations.

7 the value of teamwork.