

FOURTH SEMESTER SYLLABUS

M. Sc. MATHEMATICS



BABA GHULAM SHAH BADSHAH UNIVERSITY RAJOURI, J&K, INDIA

COURSE SCHEME

COURSE CODE	COURSE TITLE	NO. OF CREDITS	DISTRIBUTION OF MARKS		
			SA	UE	TOTAL
COMPULSORY COURSES					
MS - 431	Dissertation/ Major Project	8	50	150	200
MS - 432	Technical Communication	2	20	30	50
MS - 433	Lab course on SPSS	2	25	25	50

Choice based Complementary Electives

(Students are required to choose any **THREE** of the following courses)

MS - 434	Complex Dynamics	4	40	60	100
MS - 435	Banach Algebras	4	40	60	100
MS - 436	Advanced Functional Analysis	4	40	60	100

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MS - 437	Tensor Analysis and Riemannian Geometry	4	40	60	100
MS - 438	Algebraic Topology	4	40	60	100
MS - 439	Theory of Fields	4	40	60	100
MS - 440	Spaces of Analytic Functions	4	40	60	100
MS - 441	Algebraic Geometry	4	40	60	100
MS - 442	Theory of Relativity	4	40	60	100
MS - 443	Commutative Algebra	4	40	60	100
MS - 444	Theory of Integral Equations	4	40	60	100
MS - 445	Approximation Theory	4	40	60	100
	TOTAL	24	215	385	600

SA: Sessional Assessment

UE: University Examination

SEMESTER - IV

Course Title	Dissertation/Major Project	Maximum Marks	200
Course Code	MS-431	Supervisor	50
Credits	4	External Examiner	150

Objectives The objective of this course is to give a glimpse of research methods to the students.

- * Each student has to submit a project on a topic of his/her own choice under the supervision of a teacher selected as guide by the student's choice from the Departmental faculty or under the joint supervision of a teacher from the Department and an appropriate member from any other Department or industry but after the permission of the Departmental guide.
- * The marks by internal /external examiner will be assigned on the basis of the project report submitted by the student and the viva-voce examination.
- * The breakup for the dissertation and viva-voce marks is as follows:

	Dissertation	Viva - Voce	Total
Supervisor	30	20	50
External Examiner	100	50	150

COURSE OUTCOMES

After a student completes the Major project, we expect a student have understood.

- 1 The method of searching literature, on a particular topic, form the internet.
- 2 The various potential areas of research, in a particular field, that can lead to a research degree(M. Phil/ Ph. D).
- 3 Various ethics of good research.
- 4 How to read a research paper and present it in his / her own words.
- 5 The use of various concepts from different courses for studying a research paper.
- 6 How to employ the skills learned through different courses to simplify complicated situations.
- 7 The value of teamwork.

SEMESTER - IV

Course Title	Technical Communication	Maximum Marks	100
Course Code	MS-432	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The objective of teaching English to the students of Mathematics is to make them acquainted with English language which is now considered a global language. Acquaintance with English language will increase their prospects of employability and increase their communication skill as well.

UNIT 01 Communication-I
Scope and importance of communication; barriers to communication; verbal, non-verbal, oral and written communication; techniques to improve communication; presentation skills - effective use of presentation software and overhead, practical sessions.

UNIT 02 Communication-II
Parts of speech; words frequently misspelt; formation of words; tenses; one word substitutions; use of preposition; précis writing; narration; change of voices; paragraph writing; punctuation.

UNIT 03 Writing skills, group discussion and interview
Rules of good writing; principles of letter writing - structure and layout; curriculum vitae; letter of acceptance; letter of resignation; application / letters with bio-data; notice; agenda; minutes; group discussion - definition, methodology, helpful expression and evaluation with practical sessions; interview - types of interview and interview skills with practical session.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Be able to explain the importance of good communication skills in verbal, non-verbal, oral and written communication.
- 2 Be able to explain the techniques to improve communication and presentation skills.
- 3 Be able to write reports etc in a precise and correct way.
- 4 Be able to explain the basic principles of good writing.
- 5 Be able to explain the method of presenting one's curriculum vitae.
- 6 Be able to write various official and unofficial letters, notices, agendas, minutes of meetings etc.
- 7 Know how to behave in a group discussion with better expressions.
- 8 Know how to behave in an interview with better expressions.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Balasubramanian, T., (1981), *A Textbook of English Phonetics for Indian students*, MacMillan India Ltd.
2. Eastwood, J., (1999), *Oxford Practice Grammar*, Oxford University Press.
3. Jones, L., (1998), *Cambridge Advanced English*, Cambridge University Press.

REFERENCE BOOKS

1. Lesikar, R. V. and Pettir, Jr., (2004), *Business Communication Theory and Applications*, 6th edition, A. I. T. B. S, New Delhi.
2. Thakar, P. K., Desai, S.D. and Purani, J. J., (1998), *Developing English Skills*, Oxford University Press.

SEMESTER - IV

Course Title	Lab Course on SPSS	Maximum Marks	50
Course Code	MS-433	University Examination	25
Credits	2	Sessional Assessment	25
		Duration of Exam.	2 HOURS

Objectives The objective of this course is to introduce the basic working of the SPSS software.

- * Each student is required to maintain a practical record book.
- * Two practical tests, one Internal and one External, are to be conducted.
- * Each practical test will be of 25 marks.
- * The marks in each practical test will be divided into 4 parts, program code, program execution , Viva-Voce and practical record book.
- * The student has to pass both internal and external practical tests separately scoring a minimum of 10 marks in each test

SEMESTER - IV

Course Title	Complex Dynamics	Maximum Marks	100
Course Code	MS-434	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives	The aim of this course is to study fundamentals of complex dynamics –Conformal map, iterations of Rational functions in a plane; Fatou and Julia sets.		
UNIT 01 Conformal mapping	Linear and reciprocal transformations; square map, conformal and isogonal maps; conformality theorem; Bi-linear transformation and its basic properties; fixed points of a bilinear transformation; cross ratio; exponential and trigonometric transformations; Riemann mapping theorem (proof not included).		
UNIT 02 Iterations of functions and various metrics on \mathbb{C}_∞	Iteration of a Mobius transformation; attracting, repelling and indifferent fixed points; iterations of $R(z) = z^2$; the extended complex plane; chordal metric; spherical metric; relation between chordal metric and spherical metric.		
UNIT 03 Conjugacy classes of rational maps	Rational maps; Lipschitz condition; conjugacy classes of rational maps; valency of a function; fixed points; critical points; Riemann Hurwitz relation.		
UNIT 04 Equicontinuity and Normality	Equicontinuous functions; normality sets; Fatou sets and Julia sets; completely invariant sets; normal families and equicontinuity.		
UNIT 05 Fatou and Julia sets	Properties of Fatou and Julia sets; exceptional points; backward orbit; minimal property of Julia sets; completely invariant components of the Fatou set; the Euler characteristic.		

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concepts of repelling points, attracting points and indifferent fixed points.
- 2 Explain the concept of extended complex plane, chordal metric, spherical metric and relationship between chordal and spherical metrics.
- 3 Explain the concepts of conjugacy class of a rational map, valency of a function, completely invariant sets, normal families and equicontinuous family of functions.
- 4 Explain the concept of exceptional points, backward orbit and minimal property of Julia sets .
- 5 Explain the concepts of Fatou sets, Julia sets and relationship between them.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. **Beardon, A. F.**(1991), *Iteration of Rational Functions*, Springer Verlag, New York.
2. **Carleson, L. and Gamelin, T . W.**, (1993), *Complex Dynamics*, Springer Verlag.

REFERENCE BOOKS

1. **Hua, X. H., Yang, C. C.**, (2000), *Dynamics of Transcendental Functions*, Gordon and Breach Science.
2. **Livi, R., Nadal, J. P. and Packard, N.**, (1993), *Complex Dynamics*, Nova Science Publication, Inc.
3. **Morosawa, S., Nishimura, Y., Taniguchi, M., Ueda, T.**, (2000), *Holomorphic Dynamics*, Cambridge University Press.

SEMESTER - IV

Course Title	Banach Algebras	Maximum Marks	100
Course Code	MS-435	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to study Banach algebra and its spectral theory.

UNIT 01 Banach Algebra and its spectral properties Definition, examples and elementary properties of Banach Algebra; ideals in a Banach algebra; properties of set of invertible elements of a Banach algebra; properties of maximal ideas of a Banach algebra; quotient space of a Banach algebra; spectral of an element of a Banach algebra; formula for calculating spectral radius.

UNIT 02 Spectral and Riesz functional calculus Riesz functional calculus and its uniqueness; spectral mapping theorem; dependence of the spectral on the algebra; spectral of a linear operator; approximate point spectral of a linear operator.

UNIT 03 Abelian Banach algebra and C^* algebra Gelfand - Mazur theorem; maximal ideal space of a Banach algebra and its properties; Gelfand transforms and its properties; radical of a Banach algebra; definition, examples and elementary properties of C^* algebra; Abelian C^* - algebra and the functional calculus in C^* - algebra.

UNIT 04 C^* - algebra - I Hermitian elements; positive elements in C^* - algebra; space of positive elements and their properties; polar decomposition; ideas and quotients in C^* -algebra; representation of a c^* - algebras; cyclic representation; state of a c^* -algebra; Gelfand - Naimark - Segal construction.

UNIT 05 C^* -algebra - II Spectral measures; WOT; SOT; spectral theorem; topologies on $B(H)$; commuting operators; double commutant theorem; Fuglede - Putnam theorem; Abelian Van Neumann algebras.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concept, examples and properties of Banach Algebra, quotient space of a Banach algebra and the set of invertible elements of a Banach algebra.
- 2 Explain the concept of ideals and maximal ideals of a Banach algebra.
- 3 Explain the concept of spectrum of an element of a Banach algebra and formula for calculating spectral radius.
- 4 Explain Riesz functional calculus and its uniqueness, spectral mapping theorem and dependence of the spectral on the algebra.
- 5 Explain Gelfand - Mazur theorem, Gelfand transforms and its properties, radical of a Banach algebra and maximal ideal space of a Banach algebra with its properties.
- 6 Explain the concept and elementary properties of C^* algebra, Abelian C^* - algebra, functional calculus in C^* - algebra, positive elements in C^* - algebra and their space with properties.
- 7 Explain the concept of representation of a c^* - algebra, state of a c^* -algebra, Gelfand - Naimark - Segal construction and Abelian Van Neumann algebra.
- 8 Explain some fundamental theorems such as double commutant theorem and Fuglede - Putnam theorem.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Conway, J. B., (2008), *A Course in Functional Analysis*, 2nd edition, Springer.

REFERENCE BOOKS

1. Douglas, R. G., (2008), *Banach Algebra Techniques in Operator Theory*, 2nd edition, Springer.
2. J.M.G. Fell and R.S. Doran (1988), *Representation of *-Algebras, Locally Compact Groups and Banach * Algebraic Bundles*, Vol I, II, Academic Press.

SEMESTER - IV

Course Title	Advanced Functional Analysis	Maximum Marks	100
Course Code	MS-436	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to study advance topics of functional analysis.

UNIT 01 Topological vector spaces (TVS) Definition and examples of topological vector spaces; convex and absorbing sets; translation and multiplication operators; local base in a TVS; types of TVS; separation properties; simple properties of closure and interior in TVS.

UNIT 02 Linear transformations Continuity of linear mappings; finite dimensional spaces; relation between LCTVS and its dimension; metrization; relation between F-space and closed subspace of a TVS; bounded linear transformations; semi norm and local convexity; properties of semi norm sets; Minkowski's functional and its properties.

UNIT 03 Fundamentals theorems and special spaces Necessary and sufficient condition for a TVS to be normable; quotient spaces of a TVS; semi norm and quotient spaces; the spaces $C(\Omega)$, $H(\Omega)$; $C^\infty(\Omega)$ and Q_k , $L^p(0 < p < \infty)$; equicontinuity; Banach - Steinhaus theorem; continuity of limits of sequences of continuous linear mappings; open mapping theorem and its corollaries.

UNIT 04 Some fundamental theorems Closed graph theorem; bilinear mappings; dual space; Hahn-Banach separation theorem and its various corollaries; the weak topology of a TVS; the weak* topology of dual space of a TVS; Banach- Alaogule theorem and its applications.

UNIT 05 Convexity Convex Hull of a subset of a TVS and its properties; extreme points; the Krein- Milman's theorem; Milman's theorem; polar; bipolar theorem; Barelled and Bornological spaces; semi reflexive and reflexive topological vector spaces.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concept and examples of topological vector spaces(TVS), convex and absorbing sets, local base in a TVS and Locally convex TVS with its relation with dimension of the space.
- 2 Explain the separation properties in a TVS and the concept of closure and interior in a TVS.
- 3 Explain the concept and properties of continuity of linear mappings and relationship between F-space and closed subspace of a TVS.
- 4 Explain the concept of semi norm, its various properties and minkowski's functional.
- 5 Explain some Fundamental theorems such as Banach - Steinhaus theorem, open mapping theorem (with consequences), Closed graph theorem, Hahn-Banach separation theorem (with corollaries), Banach- Alaogule theorem (with applications), the Krein- Milman's theorem, Milman's theorem and bipolar theorem.
- 6 Explain the necessary and sufficient condition for a TVS to be normable and quotient spaces of a TVS.
- 7 Explain the spaces $C(\Omega)$, $H(\Omega)$; $C^\infty(\Omega)$ and Q_k , $L_p(0 < p < 1)$ and the continuity of limit of sequence of continuous linear mappings.
- 8 Explain the concept of bilinear mappings, the weak and weak* topology, Convex Hull (with properties), extreme points, Barelled and Bornological spaces, semi reflexive and reflexive topological vector spaces.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Rudin, W., (1973), Functional Analysis, Tata Mcgraw Hill.

REFERENCE BOOKS

1. Schwartz, L., (1975), Functional Analysis, Courant Institute of Mathematical Sciences.
2. Treves, F. (1967), Topological Vector spaces, Distributions and Kernels Academic Press.
3. Kothe, G. (1976), Topological Vector Spaces-II, Springer Verlag, New York.
4. Larsen, R., (1972), Functional Analysis, Marcel Dekker.

SEMESTER - IV

Course Title	Tensor Analysis and Riemannian Geometry	Maximum Marks	100
Course Code	MS-437	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to study fundamental ideas of tensors and their various types in detail.

UNIT Tensors
01 Idea of differentiable manifolds with n-dimensions; space of n dimensions, subspaces; transformation of coordinates; scalar; contravariant (tangent) and covariant(cotangent) vectors; scalar product of two vectors; tensor space of rank more than one contravariant and covariant tensors; symmetric and skew-symmetric tensors; addition and multiplication of tensors; contraction; composition of tensors; quotient law; reciprocal symmetric tensors of the second order.

UNIT Tensors and vectors
02 Riemannian space; fundamental tensor; length of a curve; magnitude of a vector; associated covariant and contravariant vectors; inclination of two vectors, orthogonal vectors; coordinate hypersurfaces; coordinate curves; field of normals to a hypersurface; principle directions for a symmetric covariant tensor of the second order; Euclidean space of n dimensions.

UNIT Derivative of a vector and tensor
03 Levi-Civita tensors; Christoffel symbols and second derivatives; need for covariant derivative; parallel transformations; covariant derivative of a contravariant and covariant vector; curl of a vector and its derivative; covariant differentiation of a tensor; divergence of a vector.

UNIT Geodesic
04 Gaussian curvature; Riemann curvature tensor; geodesics; differential equations of geodesics; geodesic coordinates; geodesic deviation; Riemannian coordinates; geodesic in Euclidean space; straight lines.

UNIT Tensor and curvature
05 Parallel transport along an extended curve; curvature tensor; Bianchi identities; Ricci tensor; scalar curvature; killing vector field; space-time symmetries (homogeneity and isotropy); space time of constant curvature; conformal transformations.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concept of a tensor (with various operations such as addition, multiplication, composition) contravariant and covariant tensors, symmetric and skew-symmetric tensors, Levi-Civita tensors, Christoffel symbols,
- 2 Explain the idea of differentiable manifolds and contravariant (tangent) and covariant(cotangent) vectors.
- 3 Explain the Riemannian space, coordinate hypersurfaces and field of normals to a hypersur face.
- 4 Explain the principle directions for a symmetric covariant tensor of the second order.
- 5 Explain the covariant derivative of a contravariant and covariant vector and curl of a vector with its derivative.
- 6 Explain the covariant differentiation of a tensor and divergence of a vector.
- 7 Explain Gaussian curvature, Riemann curvature tensor, geodesics and its differential equations and coordinates.
- 8 Explain the Ricci tensor, space-time symmetries (homogeneity and isotropy), space time of constant curvature and conformal transformations.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Weatherburn, C. E., (1986), *An Introduction to Riemannian Geometry and Tensor Calculus*, Cambridge University Press.
2. Narlikar, J.V., (1978), *General Relativity and Cosmology*, The Mac-Millan Company of India Ltd.

REFERENCE BOOKS

1. Srivastava, S. K. & Sinha, P. K., (1998), *Aspects of Gravitational Interactions*, Nova Science publications Inc., Commack, NY.
2. Sokolnikoff, I. S., (1964), *Tensor Analysis*, I. S. John Wiley & Sons, Inc.

SEMESTER - IV

Course Title	Algebraic Topology	Maximum Marks	100
Course Code	MS-438	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to study topology in algebraic context.

UNIT Homotopy-I
01 Homotopy of paths; equivalence of path homotopy relation; product of paths and its basic properties; fundamental group of a topological space; homomorphism induced by continuous path.

UNIT Homotopy-II
02 Covering spaces; covering map examples; local homomorphism the fundamental group of circle; lifting of a map; lifting correspondence isomorphism between S^1 and Z , retraction; non-retraction theorem; Brouwer fixed point theorem for the disc.

UNIT Fundamental groups
03 Deformation retracts and homotopy type; the fundamental group of S^n ; fundamental group of some surfaces; compactness of project plane; non commutativity of fundamental group of figure eight and double torus.

UNIT Covering spaces-I
04 Equivalence of covering spaces; the general lifting lemma; relation between equivalent covering maps and conjugations of sub group; universal covering space; space without any universal covering space; existence of covering spaces; semi locally simply connected space.

UNIT Covering spaces-II
05 Covering transformation; group of covering transformation; regular covering map; orbit space; the fundamental theorem of algebra; Borsuk-Ulam theorem for S^2 the bisection theorem.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concept of Homotopy of paths, their equivalence, product and various basic properties.
- 2 Explain the concept of fundamental group of a topological space and homomorphism induced by a continuous path.
- 3 Explain the concept of a covering space, covering map examples, local homomorphism and the fundamental group of circle.
- 4 Explain some fundamental theorems such as non-retraction theorem and Brouwer fixed point theorem for the disc.
- 5 Explain the concept of Deformation retracts and homotopy type and the fundamental group of S^n with its basic properties such as non-commutativity of fundamental group of figure eight and double torus.
- 6 Explain some fundamental theorems such as the general lifting lemma, the fundamental theorem of algebra, Borsuk-Ulam theorem for S^2 and the bisection theorem.
- 7 Explain equivalence of covering spaces, relation between equivalent covering maps and conjugations of sub group, existence of covering spaces and semi locally simply connected space.
- 8 Explain the covering transformation, group of covering transformations and regular covering map.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Munkers, J.R. ,(2000), Topology, 2nd Edition, PHI.

REFERENCE BOOKS

1. Greenberg, J. M. and Harper, R. J., (1981), Algebraic Topology: A First Course, ABP.

SEMESTER - IV

Course Title	Theory of Fields	Maximum Marks	100
Course Code	MS-439	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to study the advance topics algebra in field theory.

UNIT	Finite and algebraic extension	Definition and examples of field; extension fields; finite extension; transitivity of finite extension property; algebraic element; necessary and sufficient condition for an element to be algebraic in terms of dimension of the smallest field; subfield of algebraic elements; algebraic extension and transitivity of algebraic extension property; algebraic number; transcendence of e .
01		
UNIT	Roots of polynomials and construction with straight edge and compass	Roots of a polynomial over field; remainder theorem; number of a roots a polynomial in an extension field; existence of an extension of F of an irreducible polynomial over F ; splitting field; uniqueness of splitting field; constructible real numbers and their properties; impossibility of trisecting 60° , duplicating cube and constructing a regular septagon by straight edge and compass; derivative of a polynomial; simple extension; relation between simple extension and characteristic of a field.
02		
UNIT	Galois theory	Automorphism of a field; fixed field of a group; the group $G(K,F)$; the inequality $O(G(K,F)) \leq [K:F]$; field of symmetric rational function and its properties; normal extension and its relation with splitting field, Galois group of a polynomial; fundamental theorem of Galois theory.
03		
UNIT	Solvability by radicals and Galois group over the rationals	Solvable group; commutator subgroups; relation between solvability and commutator subgroups; homomorphic image of a solvable group; non-solvability of S_n ($n \geq 5$); relation between solvability by radicals of a polynomial and solvability of the Galois group; non-solvability of polynomial of degree ≥ 5 ; Galois group; simple extension based on above topics.
04		
UNIT	Finite fields	Number of elements in a finite field; finite fields having same number of elements; existence of finite fields; group of non-zero elements of a field; roots of an irreducible polynomials over finite fields; nature of roots; relation between splitting field of two irreducible polynomials of same elements.
05		

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concept of finite extensions, algebraic elements, algebraic numbers and transcendence of e .
- 2 Explain the concept of roots of polynomial over field, remainder theorem, irreducible polynomials, splitting field, constructible real numbers and their properties.
- 3 Explain the relation between simple extension and characteristic of a field.
- 4 Explain the concept of automorphism of a field, fixed field of a group and normal extension.
- 5 Explain the concept of fundamental theorem of Galois theory, Galois group of a polynomial.
- 6 Explain the concept of solvable group, commutator subgroup, relation between solvability and commutator subgroup.
- 7 Explain the concept of radicals, radicals of a polynomial and solvability of polynomial of degree ≥ 5 .
- 8 Explain the concept of finite field, existence of a finite field and roots of irreducible polynomials over finite fields.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Herstein, I. N., (2004), Topics in Algebra, 2nd edition, Wiley Student Edition.

REFERENCE BOOKS

2. Lidl, R. and PilzG. , (2004), Applied Abstract Algebra, 2nd edition, Springer.

SEMESTER - IV

Course Title	Spaces of Analytic Functions	Maximum Marks	100
Course Code	MS-440	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to introduced the students by the concept of Fourier series and its applications.

UNIT 01 **Fourier series** Review of Fourier series, Fourier transform and its properties; convolution theorem; the inversion theorem; uniqueness theorem; Plancherel's theorem; Parseval's formula.

UNIT 02 **Fourier transform and harmonic functions** Translation invariant subspaces of L^2 ; the Banach algebra L^1 ; complex homomorphism; the complex homomorphism of L^1 Cauchy-Riemann equation; The Laplacian; Poisson kernel; the poisson integral of a L^1 function; Harnack's theorem.

UNIT 03 **Mean value property** Mean value property; the Schwarz reflection principle; boundary behavior of Poisson integrals; Poisson integrals of measures; approach regions; maximal functions; non-tangential limits; representation theorems; Arzela-Ascoli theorem.

UNIT 04 **Hardy spaces over the unit disk** Sub-harmonic functions, Hardy space $H^p(U)$ in $H^{pn}(U)$ as a Banach space, Blaschke product and its properties, Navanlinna space N , theorem of F and M Riesz, inner and outer functions factorization.

UNIT 05 **Hardy spaces over the upper-half plane** Sub-harmonic functions in the upper-half-plane, Hardy space $H^p(\mathbb{T}^+)$ over the upper half plane, Poisson integral formula; Cauchy integral formula; boundary behavior of functions in $H^p(\mathbb{T}^+)$; canonical factorization $H^p(\mathbb{T}^+)$ as a Banach space; Paley - Wiener theorem.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concept of Fourier series, Fourier transform(with properties) and some basic theorems such as convolution theorem, the inversion theorem, uniqueness theorem, Plancherel's theorem and Parseval's formula.
- 2 Explain Translation invariant subspaces of L^2 , the Banach algebra L^1 , Poisson kernel and the Poisson integral of a L^1 function, the Laplacian and some basic theorems such as Cauchy-Riemann equation, Harnack's theorem.
- 3 Explain the concept of Mean value property, maximal functions, non-tangential limits, boundary behavior of Poisson integrals and Poisson integrals of measures.
- 4 Explain some fundamental results such as the Schwarz reflection principle, representation theorems, Arzela-Ascoli theorem.
- 5 Explain the concept of sub-harmonic functions, Hardy space $H^p(U)$ and its various features such as its Banachness.
- 6 Explain Blaschke product (with properties), Nevanlinna space N , the theorem of F and M Riesz and inner and outer functions factorization.
- 7 Explain Sub-harmonic functions in the upper-half-plane, Hardy space $H^p(\mathbb{T}^+)$ over the upper half plane and its features.
- 8 Explain Poisson integral formula, Cauchy integral formula, boundary behavior of functions in $H^p(\mathbb{T}^+)$, canonical factorization $H^p(\mathbb{T}^+)$ as a Banachspace and Paley - Wiener theorem.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Duren, P. L., (1970), Theory of HP Spaces, Academic Press.
2. Rudin, W., (1987), Real and Complex Analysis, 3rd edition, McGraw Hill Book Co.

REFERENCE BOOKS

1. Carnett, J. B.,(1981), Bounded Analytic Functions, Academic Press.
2. Hoffman, K., (2009), Banach Spaces of Analytic Functions, Prentice Hall Engle wood Cliffs, New Jersey.

SEMESTER - IV

Course Title	Algebraic Geometry	Maximum Marks	100
Course Code	MS-441	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to introduce the students by the concept of algebraic geometry and its applications

UNIT Rational maps
01 Introduction; affine varieties, Hilbert's Nullstellensatz, polynomial function and maps; rational functions and maps.

UNIT Smoothness,
02 singularity and dimension Projective space; projective varieties; rational functions and morphisms; smooth points and dimension, smooth and singular points, algebraic characterizations of the dimension of a variety.

UNIT Plane curves
03 Plane cubic curves, plane curves, intersection multiplicity, classification of smooth cubics, the group structure of an elliptic curve.

UNIT Cubic surfaces
04 Cubic surfaces, the existence of lines on a cubic, configuration of the 27 lines, rationality of cubics.

UNIT Theory of curves
05 Introduction to the theory of curves, divisors on curves, the degree of a principal divisor, Bezout's theorem, linear system on curves, projective embeddings of curves.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain rational functions and maps, affine varieties and their properties.
- 2 Explain projective space and projective varieties and algebraic characterizations of the dimension of a variety.
- 3 Explain the plane cubic curves and intersection, multiplicity, classification of smooth cubics.
- 4 Explain the group structure of an elliptic curve.
- 5 Explain cubic surfaces and the existence of lines on a cubic.
- 6 Explain configuration of the 27 lines and the rationality of cubics.
- 7 Explain divisors on curves and the degree of a principal divisor
- 8 Explain the bezout's theorem and projective embeddings of curves.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Hulek, K. (translated by H. Verrill), (2003), *Elementary Algebraic Geometry–Student Mathematical Library*, vol 20, American Mathematical Society.

REFERENCE BOOKS

1. Hartshorne, R., (1977), *Algebraic Geometry*, Springer Verlag.
2. Harris, J., (1992), *Algebraic Geometry: A First Course*, Springer Verlag.
3. *Elliptic Curves*, Notes on NBHM Instructional Conference held at TIFR, (1991), Mumbai.

SEMESTER - IV

Course Title	Theory of Relativity	Maximum Marks	100
Course Code	MS-442	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to study the theory of relativity and its applications.

UNIT 01 The special theory of relativity
Inertial frames of reference; postulates of the special theory of relativity; Lorentz transformations; length contraction; time dilation; variation of mass; composition of velocities; relativistic mechanics; world events, world regions and light cone; Minkowski space-time; equivalence of mass and energy.

UNIT 02 Energy-momentum tensors
The action principle; the electromagnetic theory; energy-momentum tensors (general); energy-momentum tensors (special cases); conservation laws.

UNIT 03 General theory of Relativity
Introduction; principle of covariance; principle of equivalence; derivation of Einstein's equation; Newtonian approximation of Einstein's equations.

UNIT 04 Solution of Einstein's equation and tests of general relativity
Schwarz's child solution; particle and photon orbits in Schwarzschild space-time; gravitational red shift; planetary motion; bending of light; radar echo delay.

UNIT 05 Brans-Dicke theory
Scalar tensor theory and higher derivative gravity; Kaluza-Klein theory.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain postulates of the special theory of relativity
- 2 Explain the concept of inertial frames of reference, lorentz transformations, length contraction, time dilation, variation of mass, composition of velocities,
- 3 Explain minkowski space-time concept and equivalence of mass and energy, the idea of action principle and energy-momentum tensors (general and special cases).
- 4 Explain the conservation laws and general theory of relativity.
- 5 Explain various principles such as principle of covariance and principle of equivalence.
- 6 Explain the einstein's equation and newtonian approximation of einstein's equations.
- 7 Explain the concepts of schwarz's child solution, particle and photon orbits in schwarzschild space-time.
- 8 Explain scalar tensor theory, higher derivative gravity and kaluza-klein theory.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Narlikar, J.V., (1988), *General Relativity & Cosmology*, 2nd edition, Macmillan Co. of India Limited.
2. Pathria, R.K.,(1994), *The Theory of Relativity*, 2nd edition, Hindustan Publishing Co. Delhi.

REFERENCE BOOKS

1. Srivastava, S. K. and Sinha, K. P., (1998), *Aspects of Gravitational Interactions*, NovaScience Publishers Inc. Commack, New York.
2. Rindler, W., (1977), *Essential Relativity*, Springer-Verlag.
3. Wald, R.M., (1984), *General Relativity*, University of Chicago Press.

SEMESTER - IV

Course Title	Commutative Algebra	Maximum Marks	100
Course Code	MS-443	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The aim of this course to study ideals, modules and rings.

UNIT Ideals
01 Ring, ring homomorphism, ideals, operation on ideals, quotient rings, zero-divisors, nilpotents and units, prime and maximal ideals, local ring, Nilradical and Jacobson radical, exercises based on above topics.

UNIT Modules
02 Module homomorphism, Submodules, Quotient modules, Operation on submodules, direct sum and product of modules, Finitely generated modules; Nakayama lemma, Tensor product of modules, Exercises based on the above topics.

UNIT Localization and decomposition
03 Localization properties of localization, primary decomposition; primary ideals, uniqueness of primary decomposition, exercises based on above topics.

UNIT Integral dependence
04 Integral dependence; transitivity of integral dependence, going-Up and going down theorems, exercises based on above topics.

UNIT Noetherianrings
05 Chain condition; Noetherian and Artinian modules, Noetherian rings; Hilbert basis theorem, irreducible ideals and primary decomposition in Noetherian rings, exercises based on above topics.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Explain the concept, examples and fundamental properties of Ring, ring homomorphism, ideals, quotient rings, zero-divisors, nilpotents and units, prime and maximal ideals, local rings, Nilradical and Jacobson radicals.
- 2 Explain the concept, examples and properties of module, Module homomorphism, Sub - modules, Quotient modules, direct sum and product of modules, Finitely generated modules and Tensor product of modules.
- 3 Explain the fundamental theorems such as Nakayama lemma.
- 4 Explain the concept and properties of Localization and primary decomposition.
- 5 Explain the concept and properties of Integral dependence, transitivity of integral dependence.
- 6 Explain the some fundamental theorems such as going-Up and going down theorems, Hilbert basis theorem.
- 7 Explain the concept of Noetherian and Artinian modules, Noetherian rings, irreducible ideals and primary decomposition in Noetherian rings.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Atiyah, M. f. and Macdonald, I. G.,(1994), Introduction to Commutative Algebra, Addison-Wesley Publishing Company.

REFERENCE BOOKS

1. Eisenbud, D., (1999), Commutative Algebra; With a View Toward Algebraic Geometry Springer- Verlag, New York.,
2. Kunz,E. (1985), Introduction to Commutative Algebraic Geometry, Birkhauser. Reid, M. (1996), Undergraduate Commutative Algebraic: London Mathematical Society Student Texts, Cambridge University Press, Cambridge.

SEMESTER - IV

Course Title	Theory of Integral Equations	Maximum Marks	100
Course Code	MS-444	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The objective of this course is to introduce students to the fundamentals of Integral Equations and their Applications.

UNIT 01 Classification of integral equation
Definition and classification of integral equations; regularity conditions; special kind of kernels; integral equation with separable kernels; reduction to a system of algebraic equation; Fredholm alternate; an approximate method.

UNIT 02 Method of successive approximations
Introduction; iterative scheme; Volterra integral equation; some results about the resolvent kernel; classical Fredholm theory; the method of solution of Fredholm ; Fredholm's first theorem.

UNIT 03 Application to ordinary differential equation
: Initial value problems; boundary value problems; Dirac- delta function; Green's function approach; Green's function for nth order ordinary differential equations.

UNIT 04 Symmetric kernels
Introduction, fundamental properties of eigen values and Eigen functions for symmetric kernel, expansion in eigen function and bilinear form, Hilbert- Schmidt theorem & consequences, solution of symmetric integral equation.

UNIT 05 Singular integral equation
Introduction; the Abel integral equation; Cauchy principal value for integrals; the Cauchy type integrals; solution of the Hilbert kernels; solution of the Hilbert type singular integral equation.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 The concept and classification of integral equations and kernels.
- 2 How to solve the volterra integral equation and fredholm integral equations by different techniques.
- 3 Applications of integral equations to initial value and boundary value problems.
- 4 The concept of dirac- delta function and green's function.
- 5 The concept of eigen values and eigen functions for symmetric kernels and their fundamental properties.
- 6 The concept of bilinear form, hilbert-schmidt theorem and its consequences.
- 7 The abel integral equation, cauchy principal value for integrals and cauchy type integrals.
- 8 How to solve hilbert type singular integral equation?

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Kanwal, R. P., (1997), *Linear Integral Equations (Theory and Technique)*, 2nd edition, Academic Press Birkhauser.

REFERENCE BOOKS

1. Porter, D., and Stirling, D. S. G., (1990), *Integral Equations a Practical Treatment from Spectral Theory to Applications*, Cambridge University Press.
2. M.L. Krasnov(1971), *Problems and Exercises Integral Equations*, Mir Publication Moscow.

SEMESTER - IV

Course Title	Approximation Theory	Maximum Marks	100
Course Code	MS-445	University Examination	60
Credits	4	Sessional Assessment	40
		Duration of Exam.	3 HOURS

Objectives The main objectives of this course is to familiarize the students with the fundamentals of approximation theory

UNIT 01 Basics of Approximation Theory: Introduction, Function Spaces, Convex and Strictly Convex Norms, The best approximation, Existence and uniqueness of best approximation (Finite-dimensional subspaces, Strictly convex spaces), Examples of nonexistence etc. A Brief Introduction to: Classical approximation, Abstract approximation, Constructive approximation, etc.

UNIT 02 Approximation by Algebraic Polynomials: Approximation by Algebraic Polynomials: Uniform Approximation by Algebraic Polynomials, the First Weierstrass Theorem, the Bernstein Polynomials.

UNIT 03 Approximation by Trigonometric Polynomials: Approximation by Trigonometric Polynomials: The second Weierstrass Theorem, the Chebyshev Polynomials, Pointwise convergence and uniform convergence, Estimates with Second Order Moduli, Absolute Optimal Constants.

UNIT 04 Positive linear operators and functional: Chebyshev conditions to choose test functions, the Bohman-Korovkin Theorem, Bernstein operators; Estimates for the Bernstein Operators, Bernstein inequality, Improved Estimates, Lupas and Phillips operators (quantum and post quantum analogue). Natural density, Statistical convergence, King's type approximation.

UNIT 05 Computer aided Geometric design (CAGD): Blending (basis) functions, Bezier curves and surfaces, de-Casteljau Degree of approximation, Lipschitz classes, Different types of modulus of continuity. algorithm, Splines, B-splines, Marsden identity, B-Splines as basis functions, Degree of Spline approximation, Knot insertion, B-splines with multiple knots, Sign changes, affine invariance, Blossoming.

COURSE OUTCOMES

On successful completion of this course, we expect that a student

- 1 Should be able to explain the concepts of function Spaces, Convex and Strictly Convex Norms and the best approximation with standard examples.
- 2 Should be able to explain the concept of Classical approximation, Abstract approximation and Constructive approximation.
- 3 Should be able to explain the concepts of Approximation and uniform approximation by Algebraic Polynomials.
- 4 Should know the fundamental theorems such as The First Weierstrass Theorem, The second Weierstrass Theorem.
- 5 Should be able to explain the concepts of degree of approximation, Lipschitz classes and different types of modulus of continuity.
- 6 Should be able to explain the concepts of Natural density, Statistical convergence, and King's type approximation.
- 7 Should be able to explain the concepts of Blending (basis) functions, Bezier curves and surfaces, ,Splines, B-Splines, and Marsden identity.
- 8 Should be able to explain the concepts of Knot insertion, B-splines with multiple knots, Sign changes, affine invariance, Blossoming.

Note for Paper Setting

The question paper will be divided into two sections. Section A will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. Section B will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.

BOOKS RECOMMENDED

TEXT BOOKS

1. Hrushikesh Narhar Mhaskar, Devidas V. Pai (2000), Fundamentals of Approximation Theory, CRC Press.
2. G. G. Lorentz , Bernstein Polynomials, Chelsa Publishing Company New York.

REFERENCE BOOKS

1. N. L. Carothers, A Short Course on Approximation Theory, Department of Mathematics and Statistics, Bowling Green State University.
2. P. P. Korovkin(1960), Linear operators and approximation theory, Hindustan Publishing Corporation, Delhi.
3. M J D Powell (1981), Approximation theory and methods, (CUP, reprinted 1988) .
4. E. W. Cheney (1982), An Introduction to Approximation Theory, 2nd ed., New York: Chelsea.
5. R. DeVore, G.G. Lorentz(1993), Constructive Approximation, Springer Verlag.
6. R Goldman (2002), Pyramid Algorithms, A Dynamic Programming Approach to Curves and Surfaces for Geometric Modeling, Elsevier.
7. Radu Paltanea (2004), Approximation Theory Using Positive Linear Operators, Birkhauser, Springer.